ATTACHMENT 1

Attachment 1. Mathematica Result

```
Δ = L1 L2 - M1 M2;
A = 1/Δ * {{0, 0, Δ/C1, 0}, {0, 0, 0, Δ/C2},
    {-L2, M2, -(R1 + R2) L2, (R3 + R4) M2}, {M1, -L1, (R1 + R2) M1, -(R3 + R4) L1}};
B = 1/Δ * {{0}, {0}, {L2}, {-M1}};
MatrixForm[A]
MatrixForm[B]
R1 = 50; R2 = 4*10^-1; R3 = 4*10^-1; R4 = 50;
L1 = L2 = 41*10^-6;
M1 = M2 = 17*10^-7;
C2 = 1*10^-9;
C1 = 1*10^-9;
```

$$\begin{pmatrix} 0 & 0 & \frac{1}{C_1} & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} \\ -\frac{L_2}{L_1 L_2 - M_1 M_2} & \frac{M_2}{L_1 L_2 - M_1 M_2} & \frac{L_2 (-R_1 - R_2)}{L_1 L_2 - M_1 M_2} & \frac{M_2 (R_3 + R_4)}{L_1 L_2 - M_1 M_2} \\ \frac{M_1}{L_1 L_2 - M_1 M_2} & -\frac{L_1}{L_1 L_2 - M_1 M_2} & \frac{M_1 (R_1 + R_2)}{L_1 L_2 - M_1 M_2} & \frac{L_1 (-R_3 - R_4)}{L_1 L_2 - M_1 M_2} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ \frac{L_2}{L_1 L_2 - M_1 M_2} \\ -\frac{M_1}{L_1 L_2 - M_1 M_2} \end{pmatrix}$$

```
MatrixForm[
 Simplify[X_S = -(ω*IdentityMatrix[4]*Cos[ω t] + A*Sin[ω t]).Inverse[(ω^2 * IdentityMatrix[4] + A.A)].B]]
```

$$\begin{pmatrix} -\dfrac{100\,000\,000\,000\,000\,000\,\left(50\,400\,000\,\omega\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-7\,945\,984\,000\,000\,000\,000\,\omega^2+168\,389\,\omega^4\right)\cos[t\,\omega]+\left(-100\,000\,00\right.\right.}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,00} \\ \dfrac{170\,000\,000\,000\,000\,000\,\omega^2\left(10\,080\,000\,000\,\omega\left(-1\,000\,000\,000\,000\,000+41\,\omega^2\right)\cos[t\,\omega]+\right.}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,000} \\ -\dfrac{100\,000\,000\,\omega\left(\left(-100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000+12\,045\,984\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\omega^2-493\,596\,3\right.\right.}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,00} \\ \dfrac{170\,000\,000\,\omega^3\left(\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-8\,454\,016\,000\,000\right.\right.}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000-1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,000} \end{pmatrix}$$

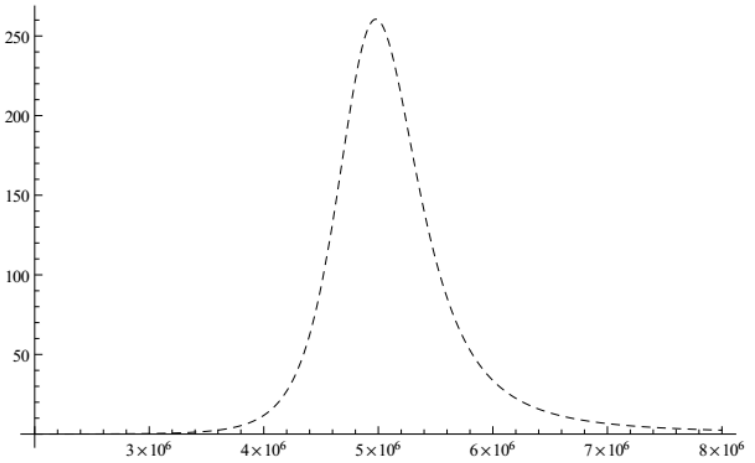
```
T = 2 π / ω;
u = Sin[ω t];
Simplify[P1 = 1/T ∫0^T Xs[[3, 1]] (u - R1 Xs[[3, 1]]) dt]
Simplify[P4 = 1/T ∫0^T R4 (Xs[[4, 1]])^2 dt]
η = P4/P1
```

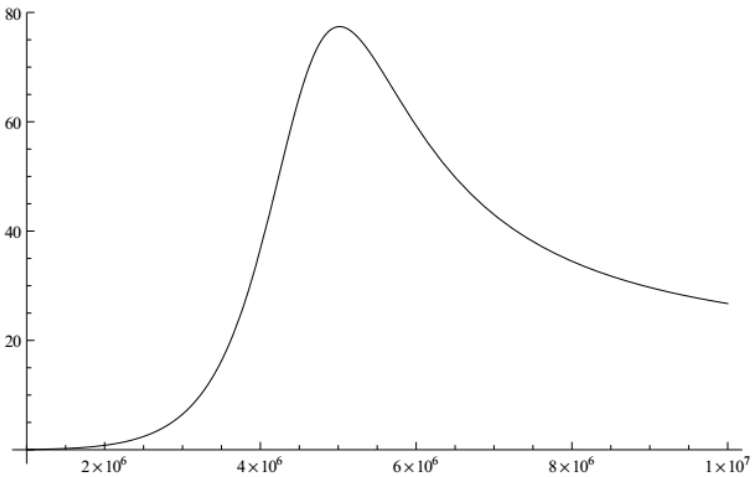
$$\left(40\,000\,000\,000\,000\,\omega^2\left(50\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 - 3\,972\,992\,000\,000\,000\,000\,\omega^2 + 102\,257\,\omega^4\right)\right) /$$
$$\left(10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 -\right.$$
$$1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\omega^2 +$$
$$\left.96\,700\,861\,728\,256\,000\,000\,000\,000\,000\,000\,000\,000\,\omega^4 - 2\,666\,553\,399\,552\,000\,000\,000\,000\,\omega^6 + 28\,160\,531\,721\,\omega^8\right)$$

$$\left(722\,500\,000\,000\,000\,000\,\omega^6\right) / \left(10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 -\right.$$
$$1\,589\,196\,800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\omega^2 +$$
$$\left.96\,700\,861\,728\,256\,000\,000\,000\,000\,000\,000\,000\,000\,\omega^4 - 2\,666\,553\,399\,552\,000\,000\,000\,000\,\omega^6 + 28\,160\,531\,721\,\omega^8\right)$$

$$\left(36\,125\,\omega^4\right) / \left(2\left(50\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 - 3\,972\,992\,000\,000\,000\,000\,\omega^2 + 102\,257\,\omega^4\right)\right)$$

```
Plot[P4 * 1 000 000, {ω, 2*10^6, 8*10^6}, PlotRange → All, PlotStyle → {Black, Dashed}]
Plot[η*100, {ω, 10^6, 1*10^7}, PlotRange → All, PlotStyle → {Black}]
```





```
FindMaximum[P4, {ω, 5*10^6}]
```

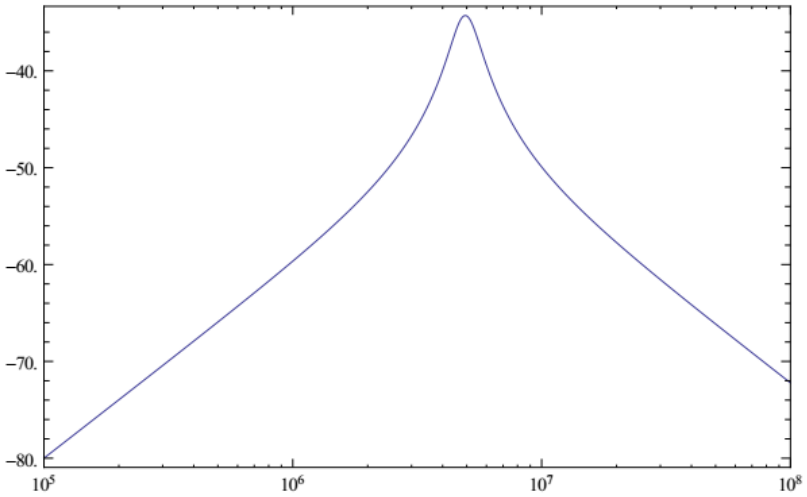
$$\{0.000260516,\ \{\omega \to 4.97829\times 10^{6}\}\}$$

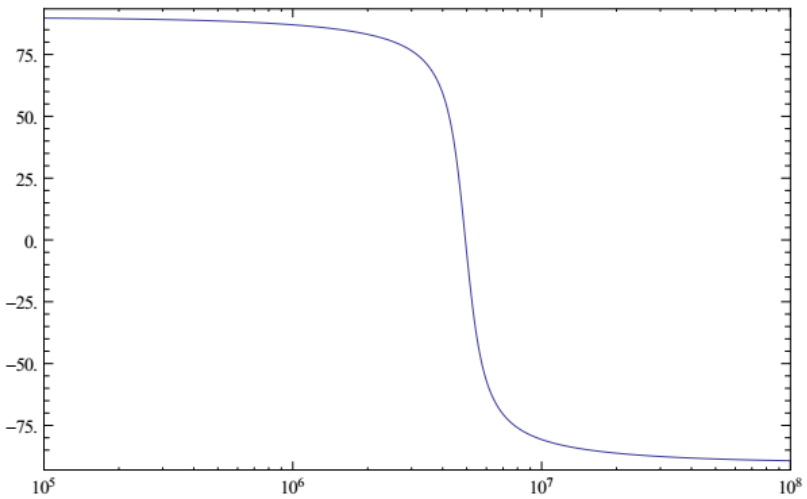
```
H1 = {{0, 0, 1, 0}};
Simplify[G1 = H1.Inverse[(s*IdentityMatrix[4] - A)].B]
H2 = {{0, 0, 0, 1}};
Simplify[G2 = H2.Inverse[(s*IdentityMatrix[4] - A)].B]
```

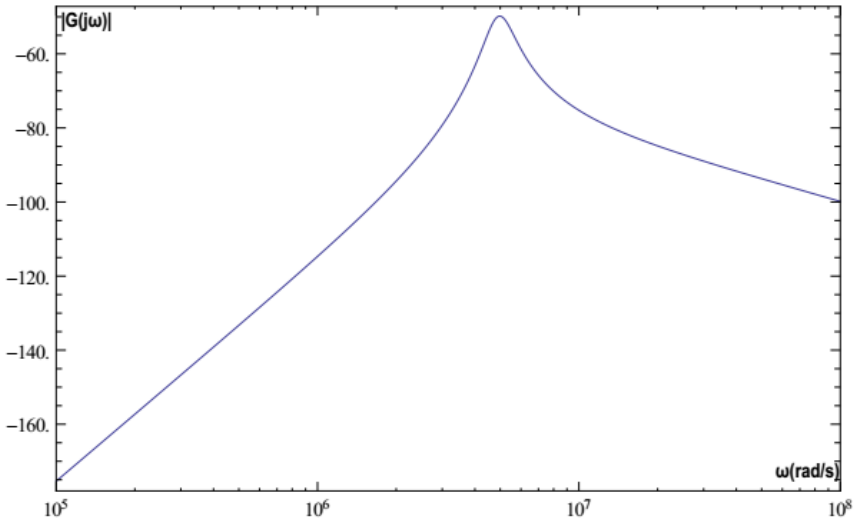
$$\{\{(100\,000\,000\,s\ (1\,000\,000\,000\,000\,000 + 50\,400\,000\,s + 41\,s^2))/(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 + 10\,080\,000\,000\,000\,000\,000\,000\,000\,s + 8\,454\,016\,000\,000\,000\,000\,s^2 + 413\,280\,000\,000\,s^3 + 167\,811\,s^4)\}\}$$

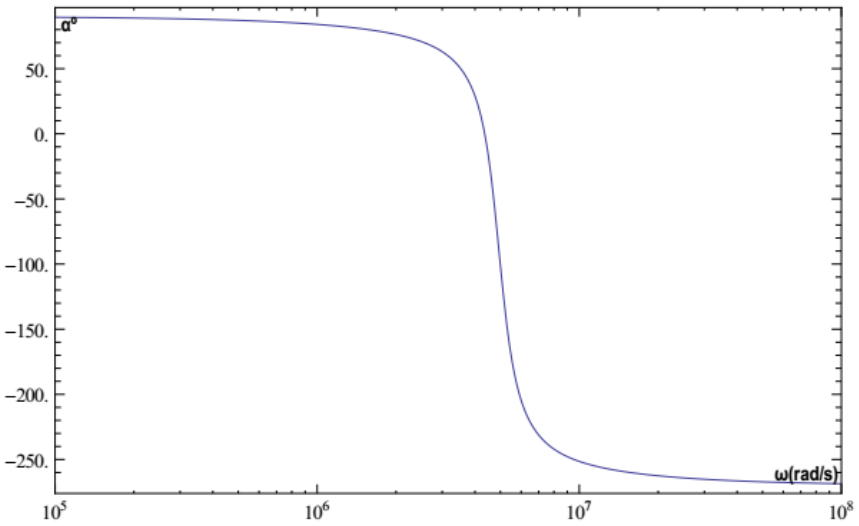
$$\{\{-(170\,000\,000\,s^3)/(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 + 10\,080\,000\,000\,000\,000\,000\,000\,000\,s + 8\,454\,016\,000\,000\,000\,000\,s^2 + 413\,280\,000\,000\,s^3 + 167\,811\,s^4)\}\}$$

```
BodePlot[G1, {10^5, 10^8}]
BodePlot[G2, {10^5, 10^8}]
```

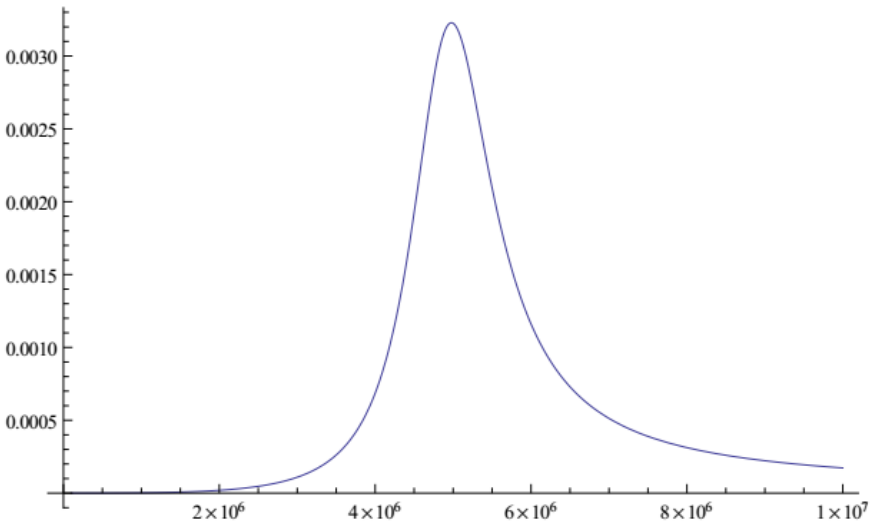






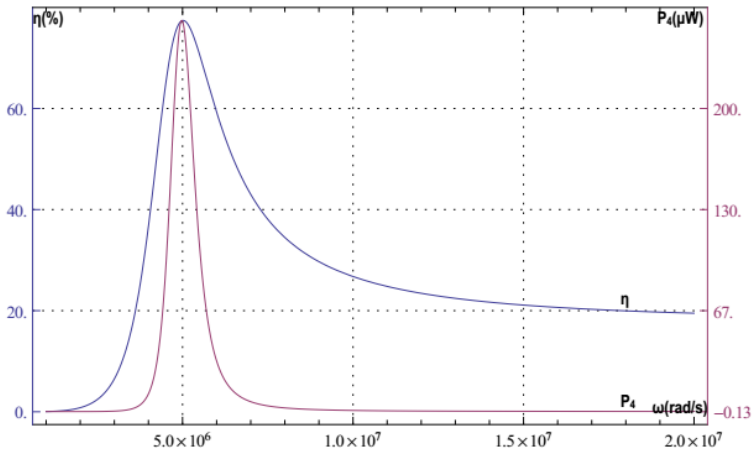



```
s = i ω;
Plot[Abs[G2], {ω, 10^5, 10^7}, PlotRange → All]
```

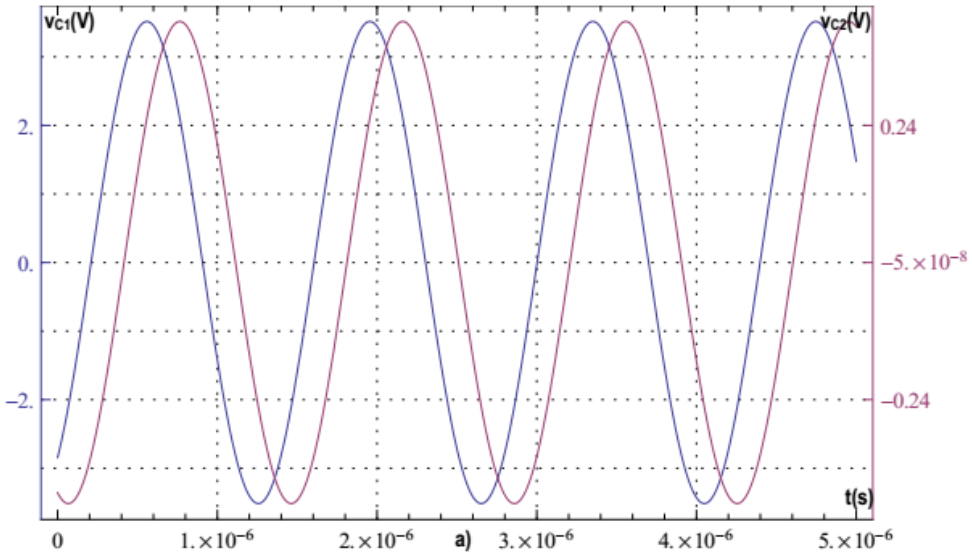


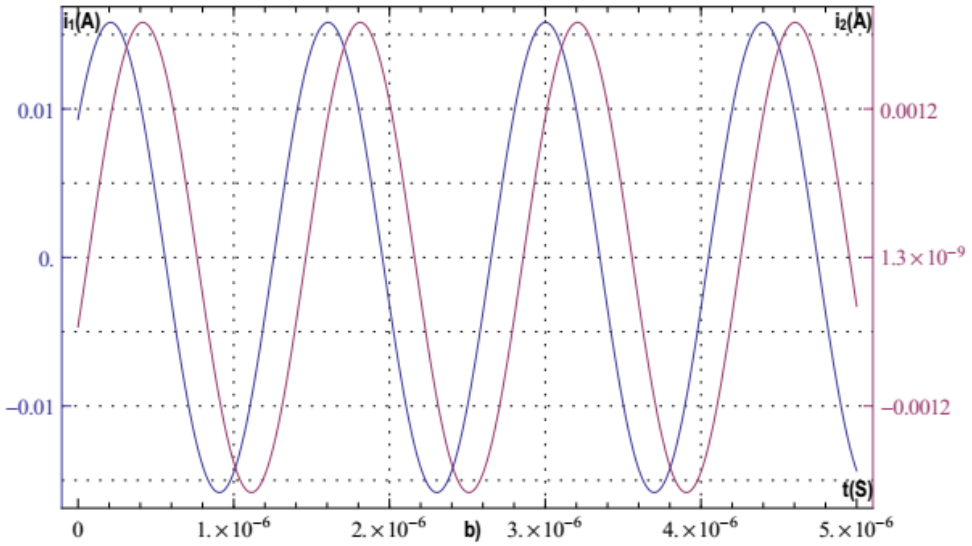
```
TwoAxisPlot[{f_, g_}, {x_, x1_, x2_}] :=
 Module[{fgraph, ggraph, frange, grange, fticks, gticks}, {fgraph, ggraph} = MapIndexed[
     Plot[#, {x, x1, x2}, Axes → True, GridLines → Automatic, GridLinesStyle → Directive[Black, Dotted],
       PlotRange → All, PlotStyle → ColorData[1][#2[[1]]]] &, {f, g}];
   {frange, grange} = (PlotRange /. AbsoluteOptions[#, PlotRange])[[2]] & /@ {fgraph, ggraph};
   fticks = N@FindDivisions[frange, 5];
   gticks = Quiet@Transpose@
      {fticks, ToString[NumberForm[#, 2], StandardForm] & /@ Rescale[fticks, frange, grange]};
   Show[fgraph, ggraph /. Graphics[graph_, s___] :→ Graphics[
       GeometricTransformation[graph, RescalingTransform[{{0, 1}, grange}, {{0, 1}, frange}]], s],
    Axes → False, Frame → True, FrameStyle → {ColorData[1] /@ {1, 2}, {Automatic, Automatic}},
    FrameTicks → {{fticks, gticks}, {Automatic, Automatic}}]]

TwoAxisPlot[{η*100, P4*1000000}, {ω, 10^6, 2*10^7}]
```



```
ω = 4.5*10^6;
TwoAxisPlot[{Xs[[1, 1]], Xs[[2, 1]]}, {t, 0, 5*10^-6}]
TwoAxisPlot[{Xs[[3, 1]], Xs[[4, 1]]}, {t, 0, 5*10^-6}]
```





$$\text{Simplify}\left[P_1 = \frac{1}{T}\int_0^T X_S[[3,1]]\,(u - R_1\,X_S[[3,1]])\,dt\right]\;//\;N$$

$$\text{Simplify}\left[P_4 = \frac{1}{T}\int_0^T R_4\,(X_S[[4,1]])^2\,dt\right]\;//\;N$$

$$\eta = \frac{P_4}{P_1}\;//\;N$$

0.000143138

0.0000923618

0.645262

ATTACHMENT 2

Attachment 2. Software Code

SCILAB

```
a=0;
b=0;
c=0;
ps1=0;
L1=0;
mu=4*%pi*10^-7;
n=1;
m=1;
r1=.025;
D1=.001;
kk=n;
ll=m;
k=n;
l=m;
for g = 1:kk
  for h = 1:ll
    a = r1+D1/2+(ll-1)*D1;
    for i = 1:k
      for j = 1:l
        b = ((r1+D1/2+(l-1)*D1)/(r1+D1/2+(ll-1)*D1));
        c = (abs((k-1)*D1-(kk-1)*D1)/(r1+D1/2+(ll-1)*D1));
        if c==0 then z=.00001; end
        R=[0,0;b,b;b,0];
        THETA=[0,0;0,%pi;%pi,%pi];
          deff('psi=f(r,theta)','psi=r*(1-r*cos(theta))*(r^2+z^2+1-2*r*cos(theta))^(-3/2)')
          [I,e]=int2d(R,THETA,f,[1.d-6, 1, 500, 40000, 1]);
        ps1=ps1+I;
      end;
    end;
    L1=L1+mu*a*ps1*10^6;
    ps1=0;
  end;
end
L1
```

PHYTON

```
import numpy as np
from scipy import interpolate, integrate, pi

mu= 4*pi*1E-7
radius_1=radius_2=25E-3
row=6
col=5
wire_1=wire_2=1E-3
dist=1E-5

d_1=d_2=wire_1
rad_1=rad_2=radius_1+.5*d_1
Z=dist
psy=0
for g in range (1,row):
   r_1 = rad_1+(g-1)*d_1
   for h in range (1,col):
      for i in range (1,row):
         r_2 = rad_2+(i-1)*d_2
         for j in range(1,col):
            z_2 = Z+(j-1)*d_2
            q=r_1/r_2
            z=z_2/r_2
            def a(r, teta): return r*(1-r*np.cos(teta))*(r**2+z**2+1-2*r*np.cos(teta))**(-3/2)
            def b(q, z): return integrate.dblquad(a, 0, pi, lambda r:0, lambda r:q)
            p, e = b(q, z)
            psy=psy+r_1*np.abs(p)

L_1= mu*psy
print(L_1)
dist=0.05
Z=dist
psy_1=0
for g in range (1,row):
   r_1 = rad_1+(g-1)*d_1
   for h in range (1,col):
      z_1 = (h-1)*d_1
      for i in range (1,row):
         r_2 = rad_2+(i-1)*d_2
         for j in range(1,col):
            z_2 = Z+z_1+(j-1)*d_2
            q=r_1/r_2
            z=z_2/r_2
            def a_1(r, teta): return r*(1-r*np.cos(teta))*(r**2+z**2+1-2*r*np.cos(teta))**(-3/2)
            def b_1(q, z): return integrate.dblquad(a_1, 0, pi, lambda r:0, lambda r:q)
            p, e = b_1(q, z)
            psy_1=psy_1+r_1*np.abs(p)

M_1=mu*psy_1
print(M_1)
```