CHAPTER 2
STUDY LITERACY

2.1 WPT

2.1.1 Biot-Savart Law

EM phenomenon has been studied for long time ago. It is a nature when electron flowing in a conductor there is magnetic field around the conductor. Figure 2. 1 shows magnetic field in the point q. Magnetic Field **B** is a vector at point q with radius r. This magnetic field magnitude is the same in around circle. Direction of magnetic field follows Fleming's Right-Hand Rule.

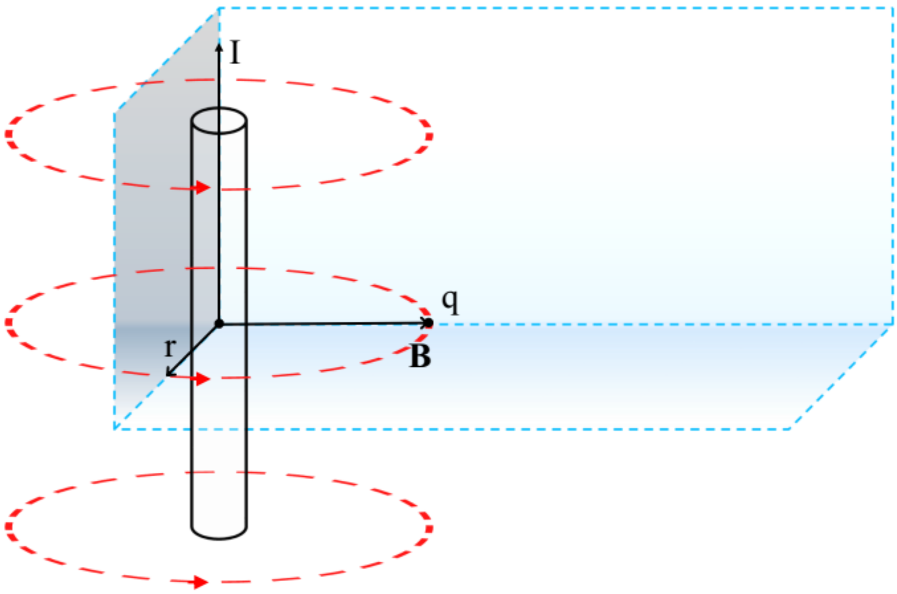

Figure 2. 1 Magnetic field around a current-carrying conductor

Biot and Savart did experiments about the force that produced by an electric current that surrounded by magnet. They concluded their experimental results in a mathematical expression. This expression gives the value of the magnetic field in space in respect with the current that produces the field. The formula is:

$$\Delta\overrightarrow{H} = \frac{i}{4\pi} \cdot \frac{\Delta l \times \vec{q}}{|q|^3} \tag{2-1}$$

Where

$\Delta\overrightarrow{H}$: slightly-different of magnetic field vector (A/m)

i : instantaneous current (A)

Δl : slightly-different of coil length (m)

$\vec{q}$: vector of examined point (m)

$|q|$: magnitude of examined point (m)

H-Field is measured in A/m, while B field is measured tesla T or W/m^2. Connection between those magnetic fields explained in Maxwell's Equation as:

$$\vec{B} = \mu\vec{H} \tag{2-2}$$

Where μ : magnetic constant (4π 10^{-7})

Applying Equation (2-1) to Equation (2-2), the equation becomes:

$$\Delta\vec{B} = \frac{\mu i}{4\pi}\cdot\frac{\Delta l\times\vec{q}}{|q|^3} \tag{2-3}$$

2.1.2 Magnetic Field Vector in Circular Current Carrying Conductor

Current flowing in circular conductor generates magnetic field in the surrounding. Since current flowing is same in any circle point, let us consider magnetic field at point P to point Q as shown in Figure 2. 2. Slightly change in coil length affects slightly change in magnetic field at Q. Plane axes in circular conductor are y-axis as radius r and x-axis as come in paper. While Z-axis is perpendicular with plane axes.

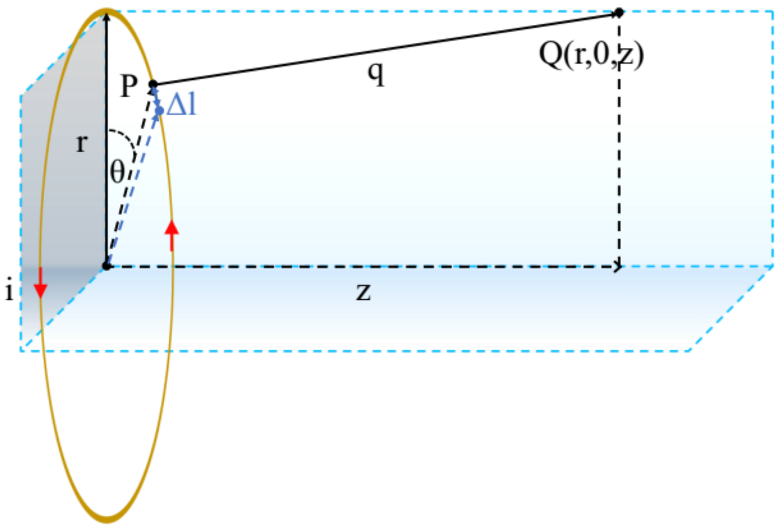

Figure 2. 2 Magnetic field on a circular current-carrying conductor

We derived the vector of conductor length and vector of magnetic point in space. Circular conductor has radius r_1 and angle θ.

$$\Delta\vec{l} = (-\sin\theta, \cos\theta, 0)r_1\Delta\theta \tag{2-4}$$

$$\vec{q} = \overrightarrow{PQ} = (x - r_1\cos\theta, -r_1\sin\theta, z) \tag{2-5}$$

$$\vec{B}=\int_0^{2\pi}\frac{\mu i_1 r_1}{4\pi}\begin{bmatrix} z\cos\theta \\ z\sin\theta \\ r-x\cos\theta \end{bmatrix}(x^2+z^2+r_1^2-2xr_1\cos\theta)^{-3/2}d\theta \tag{2-6}$$

Cosine function is even function, else sine function is odd function. The integral from 0 to 2π separated into two functions which has integral from 0 to π. Since two odd functions cancelled each other, magnetic field in y-axis is zero. Else, even function become twice of its own function.

$$B_x=\frac{\mu i_1 r_1}{2\pi}\int_0^{\pi}\cos\theta(x^2+z^2+r_1^2-2xr_1\cos\theta)^{-3/2}d\theta \tag{2-7}$$

$$B_y=0 \tag{2-8}$$

$$B_z=\frac{\mu i_1 r_1}{2\pi}\int_0^{\pi}(r_1-x\cos\theta)(x^2+z^2+r_1^2-2xr_1\cos\theta)^{-3/2}d\theta \tag{2-9}$$

2.1.3 Magnetic Field in the Z-Axes

Transmitter coil are separated with receiver coil in distance z as shown in Figure 2. 3. Magnetic field in center point of one coil to another coil is decreasing as z increasing. Considering coil 2 has slightly increase radius r and has different Δr, magnetic field is slightly change too.

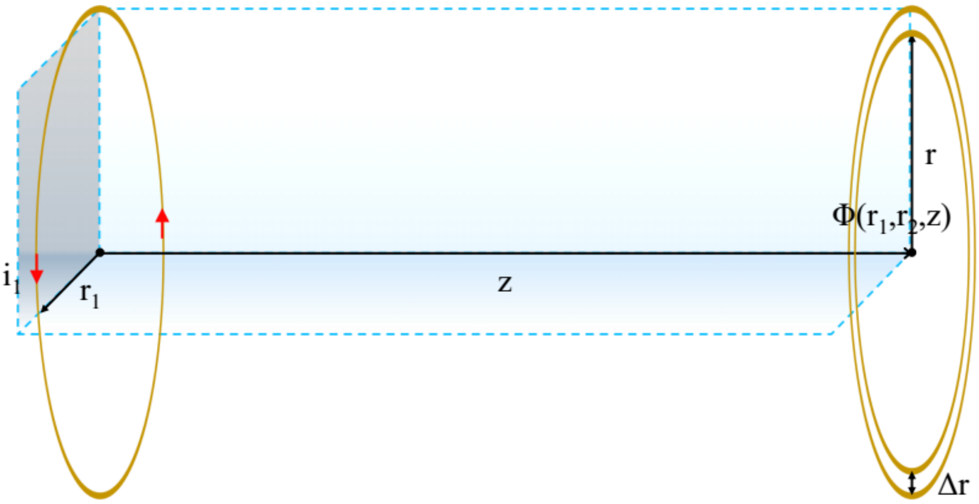

Figure 2. 3 Magnetic field of two coils in the z-axis

WPT has two coils located in the same axis. A coil itself has plane axes as an area of circle. Magnetic density defined as the integrals of magnetic field over the circle area in a coil. Magnetic field in the z-axis in Equation (2-9) is substituted in Equation (2-10). The magnetic flux φ, which is integral with the area of differential circle, is:

$$\Delta\varphi=\int\vec{B}\cdot d\vec{S} \tag{2-10}$$

$$\Delta\varphi = \left\{ \frac{\mu i_1 r_1}{2\pi} \int_0^{\pi} (r_1 - x\cos\theta)(x^2 + z^2 + r_1^2 - 2xr_1\cos\theta)^{-3/2} d\theta \right\} \cdot 2\pi r \Delta r \qquad (2\text{-}11)$$

$$\varphi = \mu i_1 r_1 \int_0^{r_2} r \int_0^{\pi} (r_1 - r\cos\theta)(r^2 + z^2 + r_1^2 - 2rr_1\cos\theta)^{-3/2} d\theta dr \qquad (2\text{-}12)$$

To get more simply magnetic flux φ, the integral parameters are divided by radius of transmitter coil r_1. If WPT has two coils with same diameter, then the ratio q is equal to 1.

$$\tilde{r} = \frac{r}{r_1}, \quad \tilde{z} = \frac{z}{r_1}, \quad q = \frac{r_2}{r_1} \qquad (2\text{-}13)$$

$$\varphi = \mu i_1 r_1 \int_0^{q} \int_0^{\pi} \tilde{r}(1 - \tilde{r}\cos\theta)(\tilde{r}^2 + \tilde{z}^2 + 1 - 2\tilde{r}\cos\theta)^{-3/2} d\theta dr \qquad (2\text{-}14)$$

Constant number which has no θ and r̃ components, are computed later to give a simple integral equation of magnetic flux φ. The ψ is the integral function that depend on q and z̃ values.

$$\psi_{(q,\tilde{z})} = \int_0^{q} \int_0^{\pi} \tilde{r}(1 - \tilde{r}\cos\theta)(\tilde{r}^2 + \tilde{z}^2 + 1 - 2\tilde{r}\cos\theta)^{-3/2} d\theta dr \qquad (2\text{-}15)$$

$$\varphi = \mu i_1 r_1 \psi_{(q,\tilde{z})} \qquad (2\text{-}16)$$

2.1.4 Magnetic Field Generated by N-Turns of Two Coils

Two coils are transmitter and receiver. One turn can produce one magnetic flux φ. Adding more turns on the coil, then the magnetic flux φ multiplies as the number of turns. Two coils in one pair also multiplies the magnetic flux.

$$\varphi = \mu i_1 r_1 n_1 n_2 \psi_{(q,\tilde{z})} \qquad (2\text{-}17)$$

2.1.5 Inductive Voltage in a Pair of Coils

Faraday observed generated voltage under magnetic field in coil edges known as induced voltage. In a pair of two coils where two currents are flowing at same directions, induced voltages are generated with polarity follows the direction of currents as shown in Fig 2. 4.

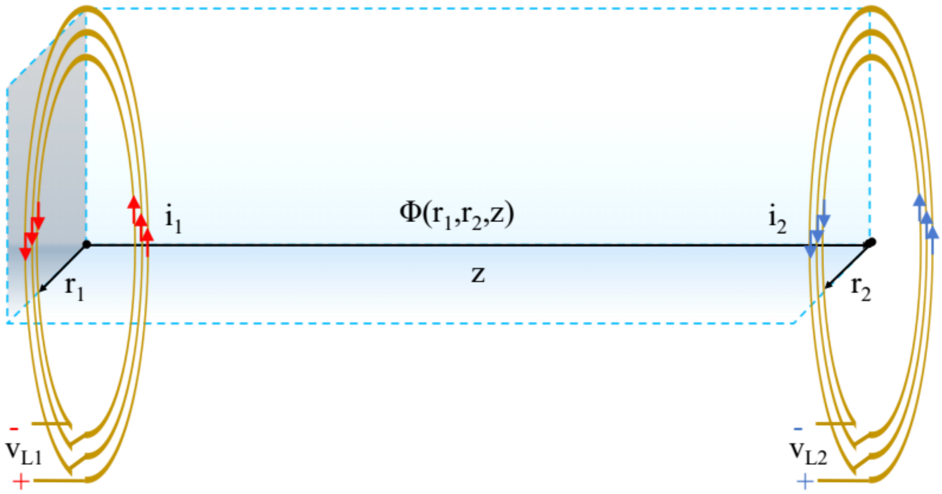

Figure 2. 4 Inductive voltage in a pair of two coils

Faraday's Law has differential form where C is the closed path encircling cross-sectional area S. Induced voltage via the time variation of a magnetic field has the potential to do work in an electric field. In this case, Faraday's Law expressed in terms of the induced electric field as:

$$\int_C E \cdot dr = -\frac{d}{dt}\left[\int_S B \cdot dS\right] \tag{2-18}$$

From that integral form, the potential of two points is the reversal of closed loop of electric field. Potential in the two point of one coil from the closed path of electric field is:

$$v_L = -\int_C E \cdot dr \tag{2-19}$$

By applying Faraday's Law in Equation (2-18) to the edge potential of one coil, the induced voltage is equal to magnetic flux φ in time rate. The equation is:

$$v_L = -\int_C -\frac{d}{dt}\left[\int_S B \cdot dS\right] = \frac{d\varphi}{dt} \tag{2-20}$$

The flux density equation is substituted from Biot-Savart in Equation (2-16). First, the voltage induced by current in same coil. The current flowing in coil 1 influence the amount of coil 1 voltage. The density flux generated by current in coil 1 induces it self. In other word, the distance z is 0. So do the coil 2 has the same condition as coil 1. The self-induced voltages v_{11} and v_{22} become:

$$v_{11} = \frac{d\varphi_{11}}{dt} = \mu r_1 n_1^2 \psi_{(1,0)} \frac{di_1}{dt} \tag{2-21}$$

$$v_{22} = \frac{d\varphi_{22}}{dt} = \mu r_2 n_2^2 \psi_{(1,0)} \frac{di_2}{dt} \tag{2-22}$$

Another induced voltage is coming from the amount of current in different coil. The current flowing in coil 2 induces the coil 1 with a finite distance z. The density flux is different from before. It does not apply z=0 anymore. The induced voltage in coil 1 because of the current flowing in coil 2, or the opposites, is:

$$v_{12} = \frac{d\varphi_{12}}{dt} = \mu r_2 n_1 n_2 \psi_{(\frac{r_1}{r_2},\frac{z}{r_2})} \frac{di_2}{dt} \tag{2-23}$$

$$v_{21} = \frac{d\varphi_{21}}{dt} = \mu r_1 n_2 n_1 \psi_{(\frac{r_2}{r_1},\frac{z}{r_1})} \frac{di_1}{dt} \tag{2-24}$$

2.1.6 Self and Mutual Inductance in a Pair of Coils

Turning a coils by hand might be difficult to arrange. Irregular turns might appear. This condition should make approach. Horizontally and vertically turns is applied as shown Fig 2. 5. Brown colored means the very first turn. Red colored is next horizontal turns notated by n. Blue colored is next vertical turns notated by m. Every horizontal turn adds distance by wire diameter between a pair of coil. While every vertical turn adds the radius between them.

Faraday's Law describes the amount of self-induction emf which is equal to the time rate of change of the magnetic flux. The magnetic flux is proportional to the magnetic field due to the source current, which in turn is proportional to the source current in the circuit. Therefore, a self-induced emf is always proportional to the time rate of change of the source current.

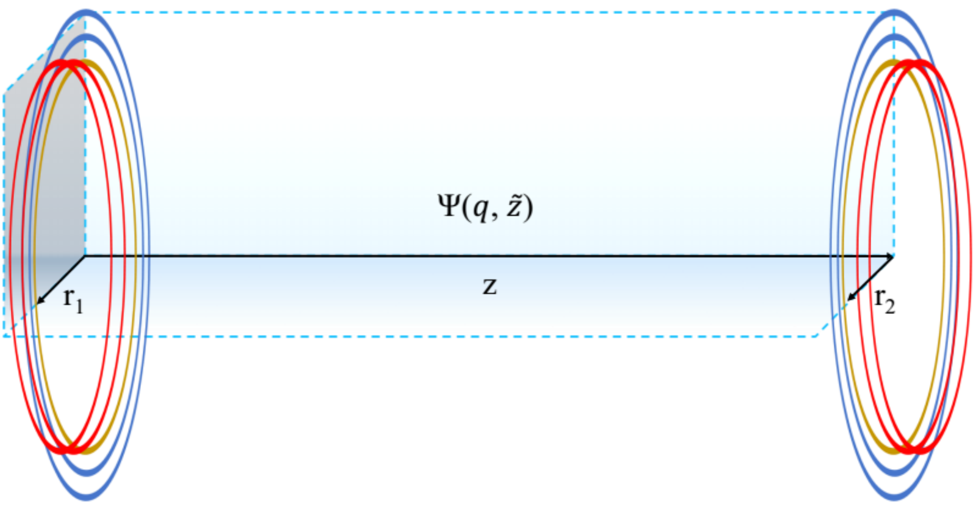

Figure 2. 5 Inductive voltage with n-turns coils

The magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits. This condition induces an emf through a process known as mutual induction, so called because it depends on the interaction of two circuits.

The induced voltage in coil 1 is coming from self-induction and another induction. In other words, the induced voltage of coil 1 v_{L1} is the sum of self-induction voltage v_{11} and another coil induction v_{12}. Coil 2 also has its self and another induction voltage. These self-inductions became constant parameters as self-inductors L_1 and L_2. Those two other inductions also constant parameters, they are mutual inductances M_{12} and M_{21}.

$$v_{L1} = v_{11} + v_{12}$$

$$v_{L2} = v_{21} + v_{22}$$

$$v_{L1} = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} \tag{2-25}$$

$$v_{L2} = M_{21} \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \tag{2-26}$$

Several turns on a coil increase amount of magnetic flux. The common way to make turn are horizontally n and vertically m. nm notation means the ratio of q or z in two pair of nm turns. Distance between two coils affects the value of magnetic flux. Far distance makes magnetic flux is slightly different in every pair of two coils. That means in far distance, magnetic flux multiply with the number of turns. In a short distance, every magnetic flux generated by a pair is different from another pair.

$$v_{11} = \mu r_1 \psi_{(1,0)} \frac{di_1}{dt} + \cdots + \mu r_1 \psi_{(q,0)} \frac{di_1}{dt} + \cdots + \mu r_1 \psi_{(nm,0)} \frac{di_1}{dt}$$

$$v_{12} = \mu r_2 \psi_{(1,\tilde{z})} \frac{di_2}{dt} + \cdots + \mu r_2 \psi_{(q,\tilde{z})} \frac{di_1}{dt} + \cdots + \mu r_2 \psi_{(nm,nm)} \frac{di_2}{dt}$$

From Equation (2-13), the ratio is changed. Now flux parameters are over r_2. The flux parameter is change with prime notation.

$$\tilde{r}' = \frac{r}{r_2},\ \tilde{z}' = \frac{z}{r_2},\ q' = \frac{r_1}{r_2}$$

$$L_1 = \mu r_1 \psi_{(1,0)} + \cdots + \mu r_1 \psi_{(q,0)} + \cdots + \mu r_1 \psi_{(nm,0)} \tag{2-27}$$

$$L_2 = \mu r_2 \psi_{(1,0)} + \cdots + \mu r_2 \psi_{(q',0)} + \cdots + \mu r_2 \psi_{(n'm',0)} \tag{2-28}$$

$$M_{12} = \mu r_2 \psi_{(1,\tilde{z})} + \cdots + \mu r_2 \psi_{(q,\tilde{z})} + \cdots + \mu r_2 \psi_{(nm,nm)} \tag{2-29}$$

$$M_{12} = \mu r_2 \psi_{(1,\tilde{z}')} + \cdots + \mu r_2 \psi_{(q',\tilde{z}')} + \cdots + \mu r_2 \psi_{(n'm',n'm')} \tag{2-30}$$

In mutual induction, the emf induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} appear to have different values, it can be shown that they are equal.

2.1.7 Ohm's Law in WPT Circuit

Employing Faraday's Law of induction, the polarities of inductor and the current directions is a fixed set as shown in Fig 2. 6. Changing current direction in revers makes polarity change. Capacitor voltage appear as parasite as well as internal resistance in a coil.

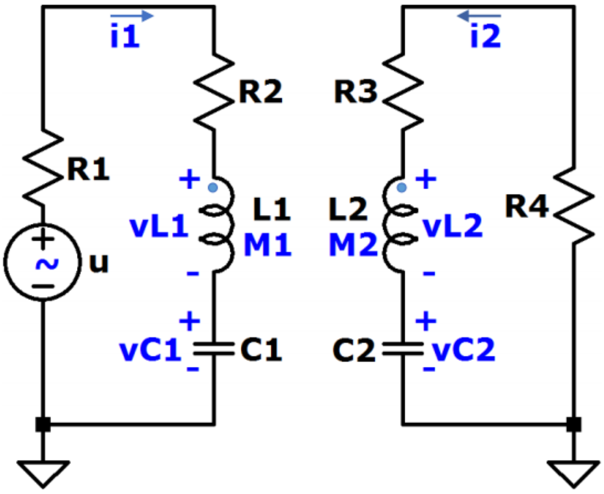

Figure 2. 6 WPT's circuit diagram

One circuit has its own equation derived from Ohm's Law and Kirchoff's Law. WPT consists of two different circuits. Transmitter circuit consists of power supply u, power supply inner resistance R_1, coil resistance R_2, self-inductance L_1, mutual inductance M_{12}, parasitic capacitance C_1. This circuit equation is:

$$u = (R_1 + R_2)i_1 + v_{C1} + L_1\frac{di_1}{dt} + M_1\frac{di_2}{dt} \tag{2-31}$$

Receiver circuit consists of load R_4, coil resistance R_3, self-inductance L_2, mutual inductance M_{21}, parasitic capacitance C_2. This circuit equation is:

$$0 = (R_3 + R_4)i_2 + v_{C2} + M_2\frac{di_1}{dt} + L_2\frac{di_2}{dt} \tag{2-32}$$

Both transmitter and receiver circuits has parasitic capacitor on it. Capacitor hold the electric energy from a electric charge flowing through it and release in a voltage form. The value of voltage depend on the integral of current in time respect. From Ohm's Law, the equations of capacitor voltages are:

$$v_{C1} = \frac{1}{C_1}\int i_1 dt \tag{2-33}$$

$$v_{C2} = \frac{1}{C_2}\int i_2 dt \tag{2-34}$$

Those four equations Equation (2-31~2-34) contain the integral differential form with time. They need to be solved into differential of i_1, i_2, v_{C1}, and v_{C2}. These four parameters are important to continue analyzing coil circuits.

2.1.8 State Space Equation

As we know, when we apply Kirchoff's Current Law (KCL) or Kirchoff's Voltage Law (KVL) in networks that contain energy−storing devices, we obtain integro−differential equations. Also, when a network contains just one such device (capacitor or inductor), it is said to be a first−order circuit. If it contains two such devices, it is said to be second−order circuit, and so on. Thus, a first order linear, time−invariant circuit can be described by a differential equation of the form [Karris, 2008]:

$$a_1\frac{dy}{dt} + a_0 y(t) = x(t) \tag{2-35}$$

A second order circuit can be described by a second−order differential equation of the same form as Equation (2-35) where the highest order is a second derivative. An nth−order differential equation can be resolved to first−order simultaneous differential equations with a set of auxiliary variables called state variables. The resulting first−order differential equations are called state−space equations, or simply state equations. These equations can be obtained either from the nth−order differential equation, or directly from the network, provided that the state variables are chosen appropriately. The state variable method offers the advantage that it can also be used with non−linear and time−varying devices. However, our discussion will be limited to linear, time−invariant circuits.

To get the equations of the circuit, Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL) are used. The equations contains integral and/or differential because of L-C components. When there are n^{th}-order differential equations, state-space equations can be obtained. The state-space equation from the circuit is:

$$\dot{x} = Ax + Bu \tag{2-36}$$

Where

$$\dot{x} = \frac{dx}{dt}$$

$$A = \frac{1}{\Delta}\begin{bmatrix} 0 & 0 & \frac{\Delta}{C_1} & 0 \\ 0 & 0 & 0 & \frac{\Delta}{C_2} \\ -L_2 & M_{21} & -(R_1+R_2)L_2 & (R_3+R_4)M_{21} \\ M_{12} & -L_1 & (R_1+R_2)M_{12} & -(R_3+R_4)L_1 \end{bmatrix}$$

$$x = \begin{bmatrix} v_{C1} & v_{C2} & i_1 & i_2 \end{bmatrix}^T$$

$$B = \frac{1}{\Delta}\begin{bmatrix} 0 \\ 0 \\ L_2 \\ -M_{12} \end{bmatrix}$$

$$\Delta = L_1L_2 - M_1M_2$$

2.1.9 Steady State Solution

If a circuit contains only one energy−storing device, the state equations are written as Equation (2-36) with output:

$$y = Cx + Du \tag{2-37}$$

Where A, B, C, D are constants matrices. Also, for two or more simultaneous differential equations, and are 2x2 or higher order matrices, and are column vectors with two or more rows. A pair of state equation Equation (2-36) and (2-37) with initial conditions:

$$x(t_0) = x_0 \tag{2-38}$$

The solution of those pair state equation is:

$$x(t) = e^{A(t-t_0)}x_0 + e^{A(t)}\int_{t_0}^{t} e^{-A(\tau)}Bu(\tau)d\tau \tag{2-39}$$

Using Equation (2-38) and substituting $u(\tau) = \sin(\omega t)$ to Equation (2-39), The Steady State Solution of Equation (2-36) is:

$$x_s(t) = -(\omega I\cos(\omega t) + A\sin(\omega t))(\omega^2 I + A^2)^{-1}B \tag{2-40}$$

2.1.10 Transfer Function

The state transition matrix can be computed from the Inverse Laplace transform. The transfer function can be found from the coefficient matrices of the state equations. The state space equation in Equation (2-36) taking Laplace transform on both sides:

$$sX(s) - x(0) = AX(s) + BU(s)$$

or

$$(sI - A)X(s) = x(0) + BU(s) \tag{2-41}$$

Multiplying both sides of Equation (41) with $(\mathrm{sI} - \mathrm{A})^{-1}$, the equation become:

$$X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}BU(s) \tag{2-42}$$

Comparing Equation (42) with:

$$x(t) = e^{A(t)}x_0 + e^{A(t)}\int_{t_0}^{t} e^{-A(\tau)}Bu(\tau)d\tau \tag{2-43}$$

The right side of Equation (2-42) is the Laplace transform of Equation (2-43). The relation between state transition matrix $e^{A(t)}$ from the Inverse Laplace of $(\mathrm{sI} - \mathrm{A})^{-1}$ is:

$$e^{A(t)} = \mathcal{L}^{-1}\{(\mathrm{sI} - \mathrm{A})^{-1}\} \tag{2-44}$$

Next, Consider output state equation is Equation (2-37):

$$y = Cx + Du$$

Taking the Laplace of both sides of Equation (2-37), the equation becomes:

$$Y(s) = CX(s) + DU(s) \tag{2-45}$$

Substituting Equation (42) to Equation (2-45), the equation becomes:

$$Y(s) = C(sI - A)^{-1}x(0) + \left[C(sI - A)^{-1}B + D\right]U(s) \tag{2-46}$$

If the initial condition is 0, Equation (2-46) reduces to

$$Y(s) = \left[C(sI - A)^{-1}B + D\right]U(s) \tag{2-47}$$

In Equation (2-47), U(s) is the Laplace transform of the input u(t); then, division of both sides by U(s) yields the transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \left[C(sI - A)^{-1}B + D\right]U(s) \tag{2-48}$$

2.1.11 Bode Plot Diagram

A Bode diagram consists of two graphs: One is a plot of the logarithm of the magnitude of a sinusoidal transfer function; the other is a plot of the phase angle; both are plotted against the frequency on a logarithmic scale.

The standard representation of the logarithmic magnitude of G(jω) is 20 log |G(jω)|, where the base of the logarithm is 10.The unit used in this representation of the magnitude is the decibel, usually abbreviated dB. In the logarithmic representation, the curves are drawn on semi log paper, using the log scale for frequency and the linear stale for either magnitude (but in decibels) or phase angle (in degrees). (The frequency range of interest determines the number of logarithmic cycles required on the abscissa.)

The main advantage of using the Bode diagram is that multiplication of magnitudes can be converted into addition. Furthermore, a simple method for sketching an approximate log-magnitude curve is available. It is based on asymptotic approximations. Such approximation by straight-line asymptotes is sufficient if only rough information on the frequency-response characteristics is needed.

The steady-state output of a transfer function system can be obtained directly from the sinusoidal transfer function, that is, the transfer function in which s is replaced by jω, where ω is frequency.

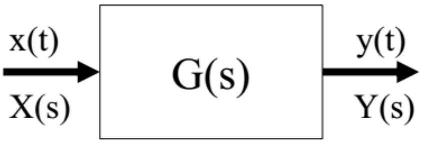

Figure 2. 7 Block diagram of a system

Consider the stable, linear time-invariant system shown in Figure 2. 7. The input and output of the system, whose transfer function is G(s), are denoted by x(t) and y(t), respectively. If the input x(t) is a sinusoidal signal, the steady-state output will also be a sinusoidal signal of the same frequency, but with possibly different magnitude and phase angle. Let us assume that the input signal is given by:

$$x(t) = X\sin(\omega t) \qquad (2\text{-}49)$$

Suppose The Laplace-transformed output Y(s) is

$$Y(s) = \frac{G(s)}{X(s)} = \frac{p(s)}{(s+s_1)(s+s_2)\cdots(s+s_n)} X(s) \qquad (2\text{-}50)$$

where X(s) is the Laplace transform of the input x(t).

After waiting until steady-state conditions are reached, the frequency response can be calculated by replacing s in the transfer function by jω. It will also be shown that the steady-state response can be given by

$$G(j\omega) = Me^{j\alpha} = M\angle\alpha \qquad (2\text{-}51)$$

where M is the amplitude ratio of the output and input sinusoids and α is the phase shift between the input sinusoid and the output sinusoid. In the frequency-response test. the input frequency ω is varied until the entire frequency range of interest is covered.

2.1.12 Average Power Input and Output

For a one-port, let the port voltage and current pair be specified as v(t) and i(t). Both of them has angular frequency with frequency f and period T.

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{2-52}$$

For a periodic voltage and current pair, the average power dissipation over a period T is defined as:

$$P_{av} = \frac{1}{T}\int_0^T v(t)i(t)dt \tag{2-53}$$

With the given WPT circuit as shown in Figure 6, the average power input P_1 and power output P_4, are:

$$P_1 = \frac{1}{T}\int_0^T i_{s1}(t)(u(t) - R_1 i_{s1}(t))dt \tag{2-54}$$

$$P_4 = \frac{1}{T}\int_0^T R_4 i_{s2}^2(t)dt \tag{2-55}$$

2.1.13 Efficiency of the WPT

Efficiency of the system is the ratio between its output with its input. Measurement of power output and input give the efficiency as follows:

$$\eta = \frac{P_4}{P_1} \tag{2-56}$$

2.2 Solar Cell

Solar cell is an electrical conversion device which transform energy from sun light into electrical energy. Conventional solar cell use photovoltaic (PV) cell. The radiation from sun light contains photon energy. When sun light reach solar cell, some energy added to material inside solar cell. Sun lights continuously add photon energy until material has enough energy to release its electron. There are many electrons released from material. This electron moves to the positive output terminal. Current will flow if the output terminal connected to load.

Solar cell output power is fluctuated. Intensity of sun light affects power output. Because of that, the control circuit should be applied to maintain the continuous electricity.

Power is define as:

$$P = IV$$

The power output of solar cell is [El-sharif]:

$$P_{out} = V_{out} I_{out} = V_{OC} I_{SC} FF \tag{2-57}$$

Where FF is coefficient factor of solar cell

Solar cell can be connected in parallel and/or series. Parallel connection of solar cell adds power output by increasing its output current. While series connection adds power output by increasing its output voltage.

The efficiency of solar cell is

$$\eta_{solar} = V_{out} I_{out} / P_{in} = V_{OC} I_{SC} FF / P_{in} \tag{2-58}$$

Solar cell can be combined in hydrogen generator. Hydrogen generator need hydrogen which produced from water electrolysis process. Solar cell produces electricity which is used to produce pure hydrogen.

2.3 Fuel Cell

Fuel Cell is electrochemical conversion device which transform hydrogen into electrical energy and vapor. In Proton Exchange Membrane (PEM), hydrogen generated by electrolysis process flows to the PEM. Proton continue flowing to other side membrane, while electron flowing to the load. On the other side membrane, proton reacted with oxygen also with electron. So that it transform into water and heat.

The decomposition chemical reaction of water is [ULUOĞLU, 2010]:

$$2H_2O(l) + electricity \rightarrow 2H_2(g) + O_2(g)$$

To decompose hydrogen and oxygen, there is minimum voltage between two electrodes. The minimum voltage V_{ref} is:

$$V_{ref} = -1.229V \tag{2-59}$$

The reaction also need external energy. The thermoneutral voltage V_{tn} is:

$$V_{tn} = -1.482V \tag{2-60}$$

The energy efficiency is given as:

$$\eta_{cell} = \frac{V_{tn}}{V_{cell}} \tag{2-61}$$

The hydrogen production need power from PV panel as shown in Figure 2. 8. Here is the data from [D. Scamman, 2014].

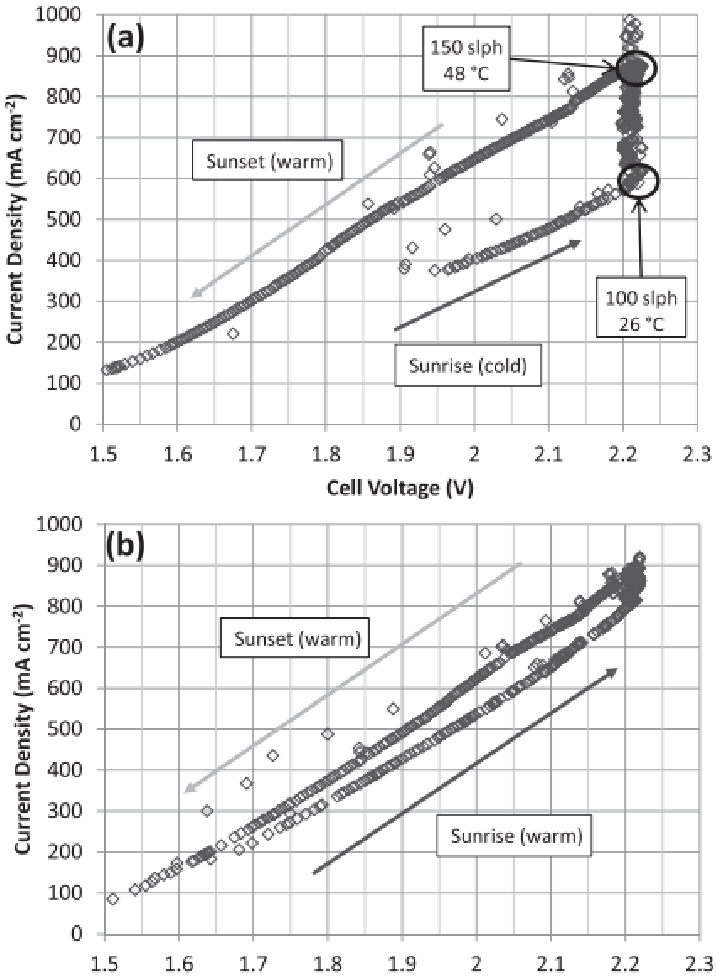

Figure 2. 8 ELY-800 polarisation curves for a) cold and b) warm start-up system temperature

Source *[Karris, 2008]*

2.4 Power Electronic Control Circuit

Electricity from solar cell depends on the availability of sun light. The sun light may be predicted but the weather can not be controlled. Sometime cloudy condition happen and reduces the intensity of sun light. Solar cell power output changes as the change of sun light intensity.

In order to get constant output voltage, solar cell output terminal should be connected to controller circuit which convert fluctuate voltage into fixed voltage. This controller circuit name SEPIC which is shown in Figure 2. 9 [Elena Niculescu, 2006].

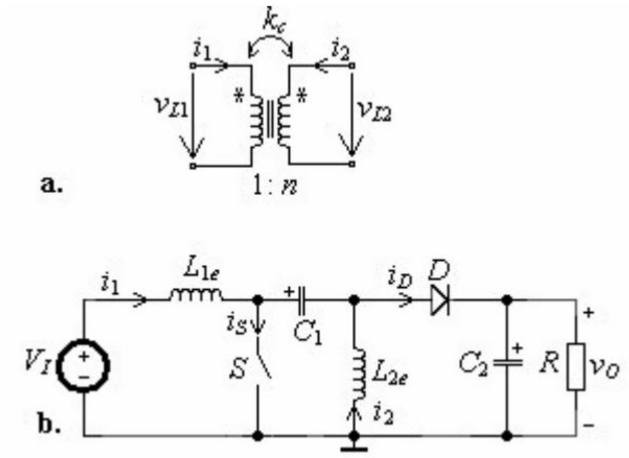

Figure 2. 9 Equivalent circuit for a) two coupled inductor and b) sepic with coupled inductor

Using the waveforms of inductor currents, the formula of average value of ripple component of inductor currents can be written as:

$$I_{L1D} = \frac{D_1(D_1 + D_2)V_1}{2L_{1e} f_s} = \frac{D_1(D_1 + D_2)V_1}{K_{1m}R} \tag{2-62}$$

$$I_{L2D} = \frac{D_1(D_1 + D_2)V_1}{2L_{2e} f_s} = \frac{D_1(D_1 + D_2)V_1}{K_{2m}R} \tag{2-63}$$

Where

I_{L1D}	: average value of i_1
I_{L2D}	: average value of i_2
D_1	: switch-on duty cycle
D_2	: diode-on duty cycle
V_I	: dc input voltage
L_{1e}, L_{2e}	: effective inductances
f_s	: switching frequency
K_{1m}, K_{2m}	: parameters of conduction through the inductors

The calculation of these components of inductor currents needs to find the parameter D_2 firstly. In order to find the expression of the component I_{L0}, we use again the relationship of the converter efficiency, that is

$$V_O I_O = \eta_{sepic} V_1 (I_{L1D} + I_{LO}) \tag{2-64}$$

For assumption of 100% efficiency, the above equation yields

$$I_{LO} = \frac{D_1 V_1}{R}\left(\frac{D_1}{K_{2m}} - \frac{D_2}{K_{1m}}\right) \tag{2-65}$$

2.5 Simulation and Mathematics Software

To help computation of formula, three kinds of mathematics software are used. They are Mathematica developed by Wolfram Research, Scilab developed by Scilab Enterprises, and Pyhthon developed by Phyton Software Foundation. One simulation software is used too. It is LT-Spice developed by Linear Technology.