

Lampiran 1. Perhitungan pembuktian syarat fungsi Lyapunov kuat untuk E^*

$$\begin{aligned}
 L'(S, I) &= \frac{dL}{dS} \cdot \frac{dS}{dt} + \frac{dL}{dI} \cdot \frac{dI}{dt} \\
 &= \frac{dS}{dt} - (\mu_3 + v + \gamma) \frac{(\mu_3 + v) + (\mu_2 + \delta - \mu_3 - v)I}{\beta S - \mu_2 - \delta + \mu_3 + v} \\
 &\quad - (\mu_3 + v + \gamma) \frac{(\mu_3 + v)S + \beta SI}{\beta S - \mu_2 - \delta + \mu_3 + v} + \beta SI \\
 &\quad - (\mu_2 + \delta + \gamma)I - I^*(\beta S - (\mu_2 + \delta + \gamma)) \\
 &= \frac{dS}{dt} - (\mu_3 + v + \gamma) \frac{(\mu_3 + v) - (\mu_3 + v)S}{\beta S - \mu_2 - \delta + \mu_3 + v} \\
 &\quad - (\mu_3 + v + \gamma) \frac{(\mu_2 + \delta - \mu_3 - v - \beta S)I}{\beta S - \mu_2 - \delta + \mu_3 + v} \\
 &\quad + (\beta S - (\mu_2 + \delta + \gamma))I - I^*(\beta S - (\mu_2 + \delta + \gamma)) \\
 &= (\mu_3 + v) + (\mu_2 + \delta - \mu_3 - v)I - (\mu_3 + v)S - \beta SI \\
 &\quad - (\mu_3 + v + \gamma) \frac{(\mu_3 + v) - (\mu_3 + v)S}{\beta S - \mu_2 - \delta + \mu_3 + v} + (\mu_3 + v + \gamma)I \\
 &\quad + \beta SI - (\mu_2 + \delta + \gamma)I - I^*(\beta S - (\mu_2 + \delta + \gamma)) \\
 &= (\mu_3 + v) - (\mu_3 + v)S \\
 &\quad - (\mu_3 + v + \gamma) \frac{(\mu_3 + v) - (\mu_3 + v)S}{\beta S - \mu_2 - \delta + \mu_3 + v} \\
 &\quad - I^*(\beta S - (\mu_2 + \delta + \gamma)) \\
 &= (\mu_3 + v)(1 - S) - (\mu_3 + v + \gamma) \frac{(\mu_3 + v - \mu_3 S - vS)}{\beta S - \mu_2 - \delta + \mu_3 + v} \\
 &\quad - I^*(\beta S - (\mu_2 + \delta + \gamma))
 \end{aligned}$$

$$\begin{aligned}
&= (\mu_3 + v)(1 - S) \left(1 - \frac{\mu_3 + v + \gamma}{\beta S - \mu_2 - \delta + \mu_3 + v} \right) \\
&\quad - I^*(\beta S - (\mu_2 + \delta + \gamma)) \\
&= (\mu_3 + v)(1 - S) \left(1 - \frac{\mu_3 + v + \gamma}{\beta S - \mu_2 - \delta + \mu_3 + v} \right) \\
&\quad - \frac{\mu_3 + v}{(\mu_3 + v + \gamma)(1 - S^*)(\beta S - (\mu_2 + \delta + \gamma))} \\
&= (\mu_3 + v)(1 - S) \left(1 - \frac{\mu_3 + v + \gamma}{\beta S - \mu_2 - \delta + \mu_3 + v} \right) \\
&\quad - \frac{\mu_3 + v}{\mu_3 + v + \gamma} (1 - S^*)(\beta S - \beta S^*) \\
&= (\mu_3 + v)(1 - S) \left(\frac{-\mu_2 - \delta + \mu_3 + v + \beta S - \mu_3 - v - \gamma}{-\mu_2 - \delta + \mu_3 + v + \beta S} \right) \\
&\quad - \frac{\mu_3 + v}{(\mu_3 + v + \gamma)(1 - S^*)(\beta S - \beta S^*)} \\
&= (\mu_3 + v)(1 - S) \left(\frac{\beta S - \beta S^*}{\beta S - \mu_2 - \delta + \mu_3 + v} \right) \\
&\quad - \frac{\mu_3 + v}{\mu_3 + v + \gamma} (1 - S^*)(\beta S - \beta S^*) \\
&= -(\mu_3 + v)\beta(S^* - S) \left(\frac{(1 - S)}{\beta S - \mu_2 - \delta + \mu_3 + v} - \frac{(1 - S^*)}{\mu_3 + v + \gamma} \right) \\
&= -(\mu_3 + v)\beta(S^* - S) \left(\frac{(1 - S)}{\beta S - \mu_2 - \delta + \mu_3 + v} - \frac{(1 - S^*)}{\beta S^* - \mu_2 - \delta + \mu_3 + v} \right) \\
&= -(\mu_3 + v)\beta(S^* - S) \\
&\quad \left(\frac{(\beta S^* - \mu_2 - \delta + \mu_3 + v)(1 - S) - (1 - S^*)(\beta S - \mu_2 - \delta + \mu_3 + v)}{(\beta S - \mu_2 - \delta + \mu_3 + v)(\beta S^* - \mu_2 - \delta + \mu_3 + v)} \right) \\
&= -(\mu_3 + v)\beta(S^* - S) \\
&\quad \left(\frac{\beta(S^* - S) + (-\mu_2 - \delta + \mu_3 + v)(S^* - S)}{(\beta S - \mu_2 - \delta + \mu_3 + v)(\beta S^* - \mu_2 - \delta + \mu_3 + v)} \right)
\end{aligned}$$

$$\begin{aligned}
 &= -(\mu_3 + v)\beta(S^* - S) \left(\frac{(S^* - S)(\beta - \mu_2 - \delta + \mu_3 + v)}{(\beta S - \mu_2 - \delta + \mu_3 + v)(\mu_3 + v + \gamma)} \right) \\
 &= \frac{-(\mu_3 + v)\beta}{(\mu_3 + v + \gamma)} \left(\frac{\beta - \mu_2 - \delta + \mu_3 + v}{(\beta S - \mu_2 - \delta + \mu_3 + v)} \right) (S^* - S)^2.
 \end{aligned}$$

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