CHAPTER 4
RESULT AND DISCUSSION

4.1 Self and Mutual Inductances

Every coil has its value of self and mutual inductance. Coil is made from copper with several turns. More turns that coil has, more magnetic flux generated. Two separated coils as one pair may be linked each other with magnetic field. One produce magnetic field another absorb it. This is a magnetic coupled of coils.

A circular coil has radius that should be considered. Biot-Savart Law explained about the circular current-carrying conductor. Magnetic field is considered from the center of the circle to a point in z-axis or finite distance. Smaller radius increase magnetic field on coil center. Also, more greater the distance, more smaller magnetic field magnitude at that point.

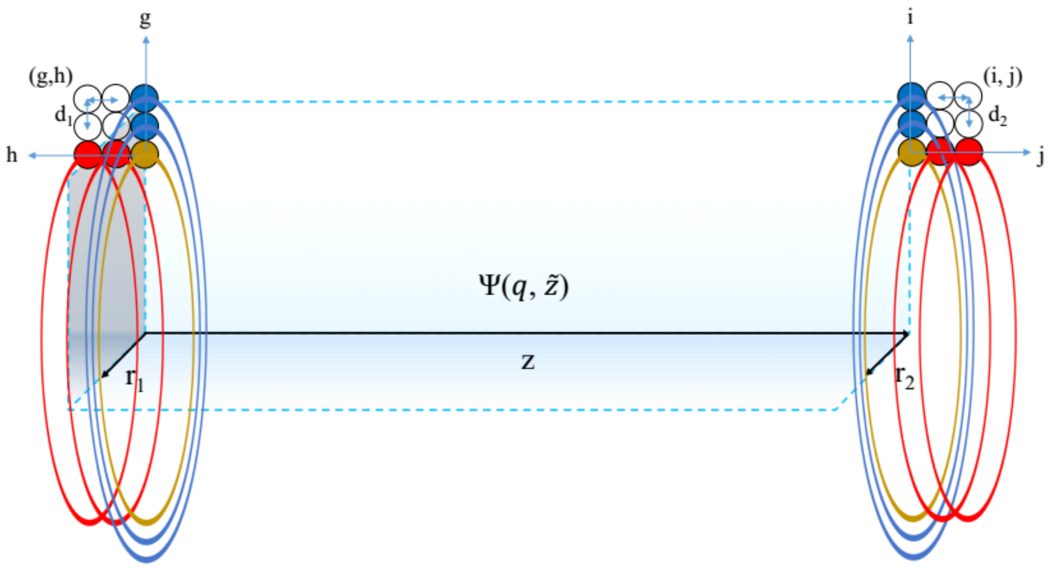

Figure 4. 1 A pair of 9-turns coils

Let two coils are identical in radius, coil diameter, and number of turns. First turns are in brown. Figure 4.1 shows each coil has nine turns which three turns in red colored are horizontal turn and others in blue colored are vertical turn. Index g and h shows row and column number of coil 1 while i and j shows that in coil 2. There is distance z between two coils and magnetic field (q, $\tilde{z}$).

To measure how much the inductances, Equation (2-27~2-30) are recalled. Numerical value of initial parameters is set. Initial parameters of coil are: H-turn = 5, V-turn = 4, Coil Cross Sectional Diameter = 1 mm, Coil Diameter = 2.5 cm, distance z = 5 cm, Copper resistivity ρ = 1.68×10^{-8} Ωm, Coil length = 3.25 m.

Then the calculation with Spyder and Mathematica are performed. Table 4.1 shows calculation results. Because two coils are identical, the value of coil 1's self-inductance L_1 is same as coil 2's self-inductance L_2. So do the coil 1's and coil 2's mutual inductances M_1 and M_2 is same.

Table 4. 1

Calculation Result of Self and Mutual Inductances

Coil Parameters	Spyder	Mathematica
L_1 and L_2	4.05029×10^{-5}	4.3513×10^{-5}
M_{12} and M_{21}	1.6978×10^{-6}	1.6643×10^{-6}

4.2 Power Input and Output

Value of self and mutual inductance have been obtained. Internal resistance and capacitance also has to be known. To find internal resistance, resistivity and dimension of coil are needed. In general, coil is made from copper which has resistivity 1.68×10^{-8} Ωm at 20 ºC. Coil length is 3.25 m. Diameter of coil is 1 mm. So, internal resistivity of coil 1 and coil 2, R_2 and R_3, are 0.4 Ω. All power supply has output impedance. This output impedance R_1 considered as 50 Ω. The electrolysis in electrode is considered as resistance load R_4 with 50Ω.

Capacitance is series connected with inductor and has very small value in a coil. The values of capacitance are needed to set the frequency of WPT. This is similar with tuning the radio capacitance to get the desired frequency modulation. Let us consider the capacitance values, C_1 and C_2, are 1 nF.

WPT can be modeled in a circuit diagram. From this diagram, the circuit equations can be derived. Since there are reactance on it, the equations should have time rate. It is just nature, since reactance can save electric energy. Capacitance able to save electric energy from electric charge, and release saved energy in voltage form. Inductance able to save electric energy from electric charge, and release saved energy in current voltage.

WPT circuit diagram is not simple as imagined. There are four components which has relations with time rate, they are two inductors and two capacitors. They have their own current and voltage. Since the two currents and two voltages are needed to calculate power input as well as power output, we take four different equation. This equation must represent currents and voltages in two coils. That is defined as a function $x = [v_{C1} \quad v_{C2} \quad i_1 \quad i_2]^T$ with function time rate is $\dot{x} = \frac{dx}{dt}$.

When there are multiple differential equations, state space equation can be derived. In control theory, state space equation very convenient to find transfer function. It also able to calculate the steady state equation. It is similar to this WPT. The state equation of WPT circuit is derived in Equation (2-36). The steady state solution with AC power supply is derived in Equation (2-40). This equations represent steady state currents and voltages in both coil.

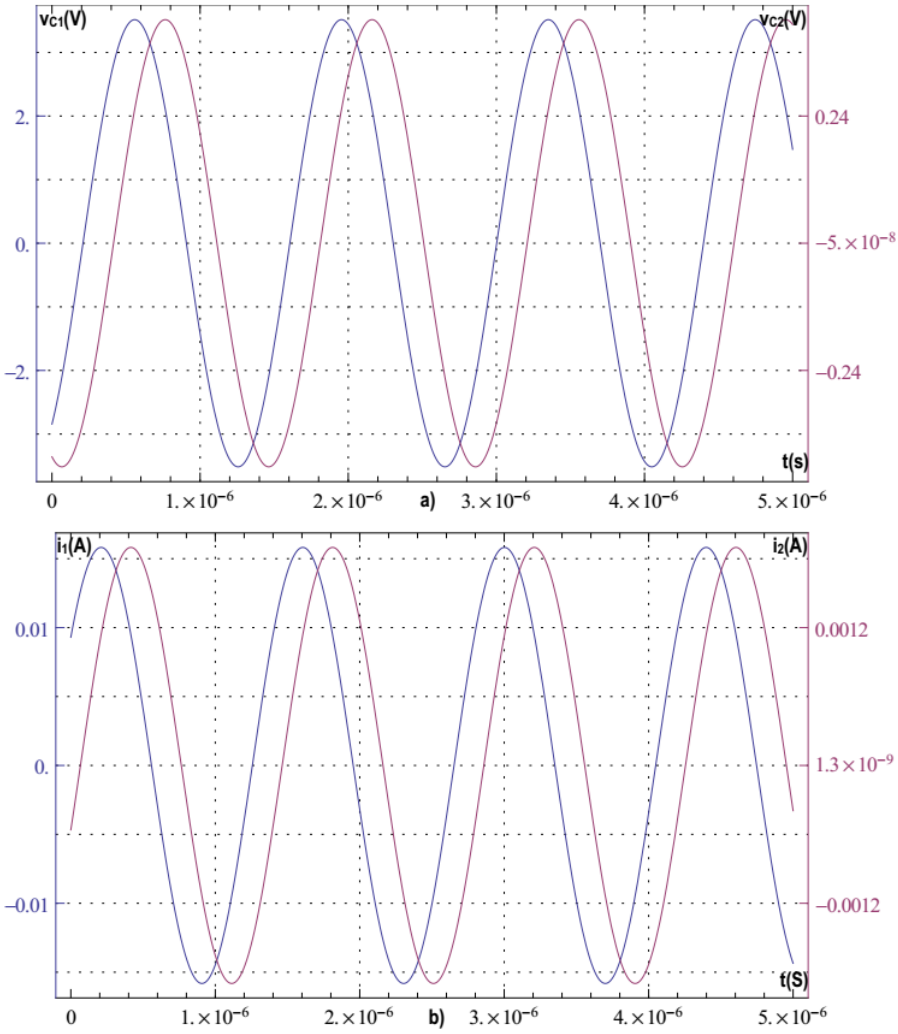

Figure 4. 2 Mathematica's steady state waveform of a) voltages and b) currents

Defining the angular frequency with $\omega = 4.5 \times 10^{-6}$ rad/s, the steady state equation in respect with times can be obtained as in Figure 4.2. It can be inferred that amplitude of v_{C1} and i_1 is bigger than amplitude of v_{C2} and i_2. Fig 4.3 which is simulating WPT with LT-Spice is similar to mathematica result. The appearance of inductors bigger than capacitors makes the waveforms between coil 1 and coil 2 lagged. At $\omega = 5 \times 10^{-6}$ rad/s, using Equation (2-54) and (2-55), power input P_1 is 0.335851 mW and power output P_4 is 0.25994 mW.

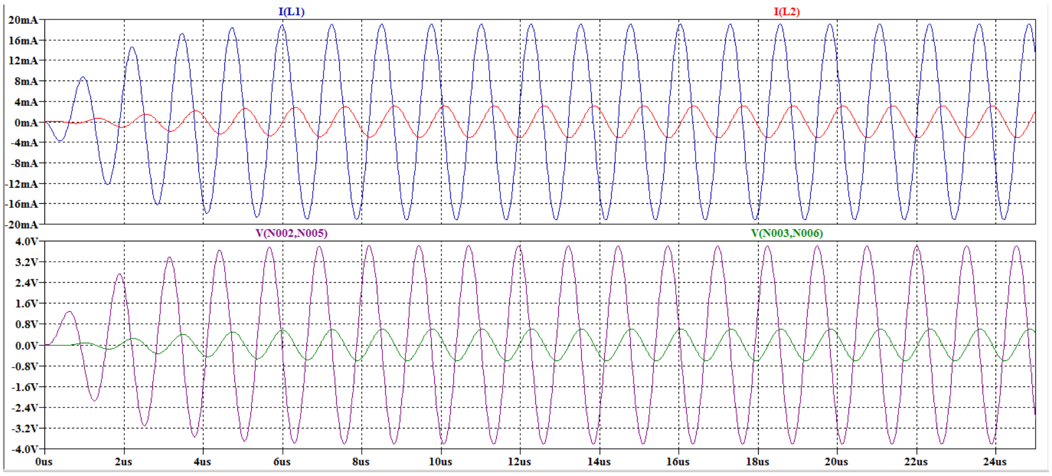

Figure 4. 3 Voltages and currents with LT-Spice simulation

4.3 Efficiency

Efficiency is the ratio between power output and power input as stated in Equation (2-56). The value of efficiency η with $\omega = 5 \times 10^{-6}$ rad/s, is 77.3975 %.

4.4 Resonance Frequency

AC power supply has frequency in its waveform. Some device need low frequency and some need high frequency. Electricity generated in mesh use low frequency 50 or 60 Hz. While communication system use high frequency in many ranges such as kHz and MHz. There is difference capacity of carrying power, low frequency able to carry more power than high frequency.

Considering frequency at WPT is important whether it produce more power or less power. Slightly change of frequency may decrease or increase power. To analyze a system with the frequency domain, frequency response analysis with bode plot should be employed. From state-space equation above, and using Equation (2-48), gain and phase can be obtained. Matrix D is initial condition of v_{C1}, v_{C2}, i_1, and i_2. If there is no energy saved in capacitors and inductors then value of D is 0.

Gain is s-function which s is equal to iω. Since there is imaginary on s value, gain value is in complex number. This complex number has magnitude and phase in polar form. Bode Plot shows the connection both of them in the angular frequency axis. Figure 4.4 shows the gain of WPT system. Minus sign of the gain means input is bigger than output. While minus sign in phasor is output wave is being shifted. The angular frequency ranged between 10^5 until 10^8 rad/s. That bode plot helps us to find at which frequency has the highest gain.

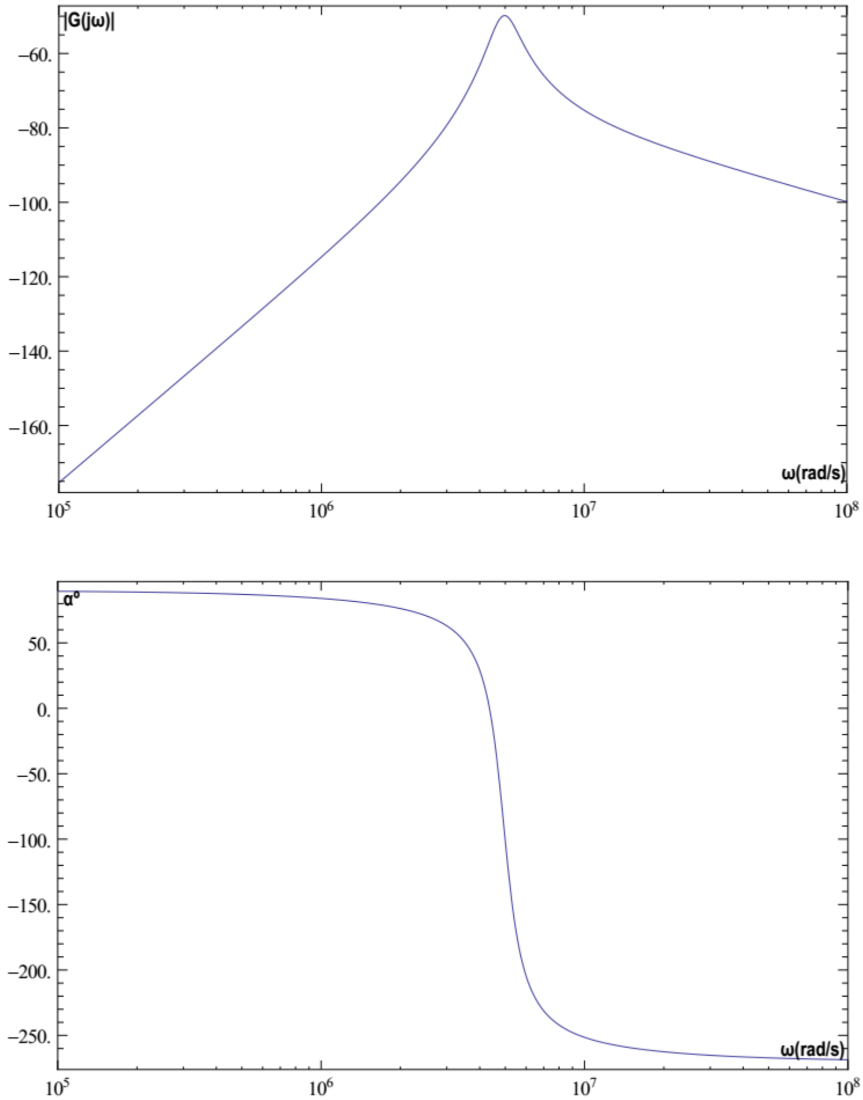

Figure 4. 4 Mathematica's bode plot of the wpt system

There is peak where gain is in highest value. The frequency which makes WPT produces high gain is resonance frequency. When resonance happens in a circuit, the value of reactance become zero. In other words, impedance of capacitor and inductor are cancelled each other. Figure 4.5 shows bode plot with LT-Spice. It shows power output gain and phase with respect with frequency ranged from 100 kHz to 10 MHz.

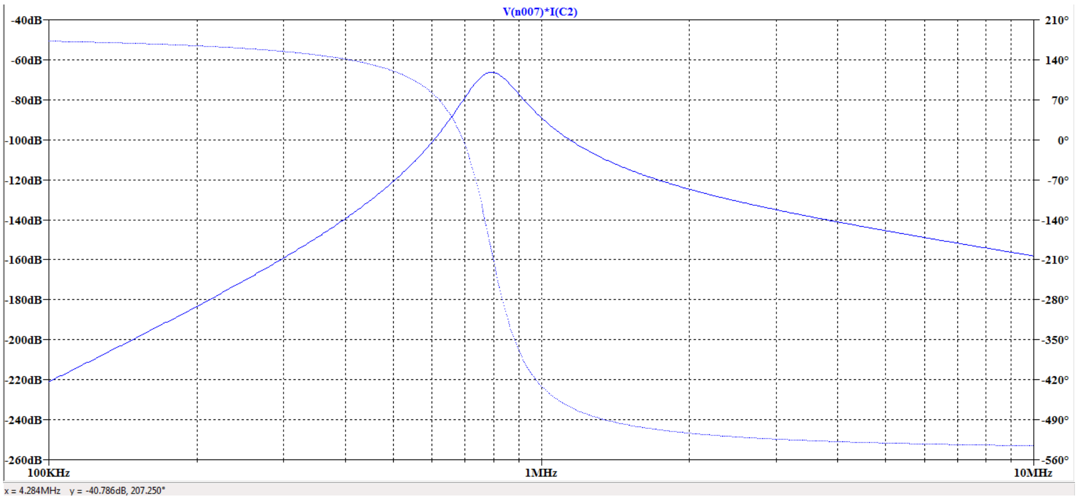

Figure 4. 5 LT-Spice's bode plot of the wpt system

It can be inferred from Figure 4.6 that angular frequency which resonance is happening is $\omega = 5 \times 10^{-6}$ rad/s. If angular frequency changed become 4.5×10^{6}, then power input P_1 is 0.143138 mW, power output is P_4 is 0.0923618 mW, and efficiency η is 64.5262 %.

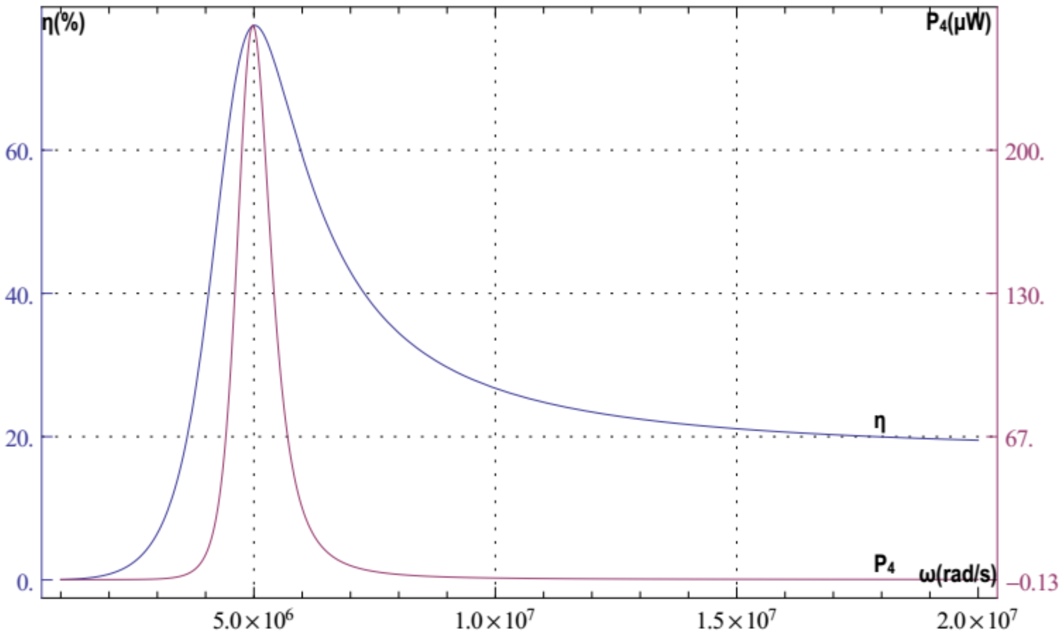

Figure 4. 6 Power output and efficiency of the wpt