LAMPIRAN

Lampiran 1 Perhitungan Titik Kesetimbangan Endemi

Titik kesetimbangan sistem persamaan (3.4) diperoleh jika

$$\dot{S} = \dot{I},$$

$$a - \frac{\beta S I}{S + I} - bS + dI + fQ - \frac{\gamma(1-\theta_d)\alpha SI}{S + (1-\theta_d)I}$$
$$= \frac{\beta S I}{S + I} - (c + d + \alpha)I + (1-\theta_e)(1-\theta_d)\alpha I$$
$$- \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha SI}{S + (1-\theta_d)I}$$

$$a - \frac{\beta S I}{S + I} - bS + dI + fQ - \frac{(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} - \frac{\beta S I}{S + I}$$
$$+ (c + d + \alpha)I - (1-\theta_e)(1-\theta_d)\alpha I$$
$$+ \frac{(1-\theta_e)(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} = 0$$

$$a - bS + fQ + dI - \frac{(1-\theta_d)\gamma\alpha SI}{S + (1-\theta_d)I} - \frac{2\beta S I}{S + I} + (c + d)I + \alpha I - \alpha I$$
$$+ \theta_d\alpha I + \theta_e(1-\theta_d)\alpha I - \frac{(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I}$$
$$+ \frac{\theta_e(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} = 0$$

$$cI + eQ + fQ + dI - \frac{2(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} - \frac{2\beta S I}{S + I} + (c + d)I + \theta_d\alpha I$$
$$+ \theta_e(1-\theta_d)\alpha I + \frac{\theta_e(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} = 0$$

$$(e + f)Q - \frac{2(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} - \frac{2\beta S I}{S + I} + 2(c + d)I + \theta_d\alpha I$$
$$+ \theta_e(1-\theta_d)\alpha I + \frac{\theta_e(1-\theta_d)\alpha\gamma SI}{S + (1-\theta_d)I} = 0$$

$$\theta_d \alpha I + \theta_e(1-\theta_d)\alpha I + \frac{\theta_e(1-\theta_d)\alpha\gamma SI}{S+(1-\theta_d)I} - \frac{2(1-\theta_d)\alpha\gamma SI}{S+(1-\theta_d)I} - \frac{2\beta S I}{S+I} + 2(c+d)I + \theta_d \alpha I + \theta_e(1-\theta_d)\alpha I + \frac{\theta_e(1-\theta_d)\alpha\gamma SI}{S+(1-\theta_d)I} = 0$$

$$2(c+d)I + 2\theta_d \alpha I + 2\theta_e(1-\theta_d)\alpha I + \frac{2\theta_e(1-\theta_d)\alpha\gamma SI}{S+(1-\theta_d)I} - \frac{2(1-\theta_d)\alpha\gamma SI}{S+(1-\theta_d)I} - \frac{2\beta S I}{S+I} = 0$$

$$(c+d+\theta_d\alpha + \theta_e(1-\theta_d)\alpha)I + (\frac{\theta_e(1-\theta_d)\alpha\gamma}{S+(1-\theta_d)I} - \frac{(1-\theta_d)\alpha\gamma}{S+(1-\theta_d)I} - \frac{\beta}{S+I})S I = 0$$

$$(c+d+\theta_e\alpha + (1-\theta_e)\theta_d\alpha)I - (\frac{(1-\theta_e)(1-\theta_d)\alpha\gamma}{S+(1-\theta_d)I} + \frac{\beta}{S+I})S I = 0$$

$$(c+d+\theta_e\alpha + (1-\theta_e)\theta_d\alpha)I - (\frac{(1-\theta_e)(1-\theta_d)\alpha\gamma}{S+(1-\theta_d)I} + \frac{\beta}{S+I})S I = 0 \qquad (1.11)$$

Dari persamaan (1.11) dapat dimisalkan,
$\eta = c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha,\ \psi = (1-\theta_e)(1-\theta_d)\alpha\gamma.$
sehingga persamaan (1.1) menjadi

$$\eta I - (\frac{\psi}{S+(1-\theta_d)I} + \frac{\beta}{S+I})S I = 0.$$

$$\eta I - (\frac{\psi(S+I) + \beta(S+(1-\theta_d)I)}{(S+I)(S+(1-\theta_d)I)})S I = 0$$

$$\eta I(S+I)(S+(1-\theta_d)I) - (\psi(S+I) + \beta(S+(1-\theta_d)I))S I = 0$$

$$\eta I(S^2 + SI + (1-\theta_d)SI + (1-\theta_d)I^2) - (\psi S + \psi I + \beta S + \beta(1-\theta_d)I)S I = 0$$

$$I(\eta S^2 + \eta SI + \eta(1-\theta_d)SI + \eta(1-\theta_d)I^2) - (\psi S^2 + \psi SI + \beta S^2 + \beta(1-\theta_d)SI)I = 0$$

$$(\eta S^2 + \eta SI + \eta(1-\theta_d)SI + \eta(1-\theta_d)I^2 - \psi S^2 - \psi SI - \beta S^2 - \beta(1-\theta_d)SI)I = 0$$

$$\eta S^2 + \eta SI + \eta(1-\theta_d)SI + \eta(1-\theta_d)I^2 - \psi S^2 - \psi SI - \beta S^2 - \beta(1-\theta_d)SI = 0$$

$$-\eta S^2 - \eta SI - \eta(1-\theta_d)SI - \eta(1-\theta_d)I^2 + \psi S^2 + \psi SI + \beta S^2 + \beta(1-\theta_d)SI = 0$$

$$(\beta + \psi - \eta)S^2 + (\psi + \beta(1-\theta_d) - \eta - (1-\theta_d)\eta)SI - \eta(1-\theta_d)I^2 = 0$$

$$(\beta + \psi - \eta)S^2 + (\psi + \beta - \beta\theta_d - \eta - (1-\theta_d)\eta)SI - \eta(1-\theta_d)I^2 = 0$$

$$(\beta + \psi - \eta)S^2 + (\psi + \beta - \beta\theta_d - \eta - (1-\theta_d)\eta)SI - \eta(1-\theta_d)I^2 = 0 \qquad (1.12)$$

Dari persamaan (1.12) dapat dimisalkan bahwa $\xi = \beta + \psi - \eta$, sehingga persamaan tersebut menjadi

$$\xi S^2 + (\xi - (1-\theta_d)\eta - \beta\theta_d)SI - \eta(1-\theta_d)I^2 = 0. \qquad (1.13)$$

Persamaan (1.13) dimisalkan lagi untuk menyederhanakan perhitungan yaitu

$$\omega = \xi - (1-\theta_d)\eta - \beta\theta_d.$$

Akar dari persamaan (1.13) dengan mengabaikan nilai I^* maka didapat

$$M = \frac{-(\omega) + \sqrt{\omega^2 + 4\xi\eta(1-\theta_d)}}{2\xi}$$

jadi $S^* = MI^*$.

Nilai $S^* = MI^*$ disubtitusikan ke persamaan (3.7c) didapat

$$\theta_d \alpha I^* + \theta_e(1-\theta_d)\alpha I^* + \frac{\theta_e(1-\theta_d)\alpha\gamma S^* I^*}{S^* + (1-\theta_d)I^*} - (e+f)Q = 0$$

$$(e+f)Q = \theta_d \alpha I^* + \theta_e(1-\theta_d)\alpha I^* + \frac{\theta_e(1-\theta_d)\alpha\gamma S^* I^*}{S^* + (1-\theta_d)I^*}$$

$$Q^* = \frac{1}{e+f}(\theta_d\alpha + \theta_e(1-\theta_d)\alpha + \frac{\theta_e(1-\theta_d)\alpha\gamma M}{M + (1-\theta_d)I^*})I^*.$$

Dimisalkan $N = \frac{1}{e+f}(\theta_d\alpha + \theta_e(1-\theta_d)\alpha + \frac{\theta_e(1-\theta_d)\alpha\gamma M}{M+(1-\theta_d)I^*})$

sehingga $Q^* = NI^*$

jika persamaan (3.4a), (3.4b) dan (3.4c) dijumlahkan maka didapat persamaan

$$a - bS - cI - eQ = 0 \tag{3.14}$$

Nilai $Q^* = NI^*$ disubsitusikan ke dalam persamaan (3.14) sehingga diperoleh

$$a - bMI - cI - eNI = 0$$
$$a = (bM + c + eN)I$$
$$I^* = \frac{a}{bM + c + eN}.$$

jadi titik kesetimbangan endeminya yaitu

$$E_+ = (S^*, I^*, Q^*, S^*, I^*, Q^*)$$

dengan $S^* = M(\frac{a}{bM+c+eN})$, $Q^* = N(\frac{a}{bM+c+eN})$, $I^* = \frac{a}{bM+c+eN}$

Lampiran 2 Perhitungan Matriks Generasi Selanjutnya

Matriks k dan matriks y adalah

$$k = \begin{pmatrix} \dfrac{\beta S_1 I_1}{S_1 + I_1} + \dfrac{(1-\theta_d)(1-\theta_e)\gamma\alpha S_2 I_2}{S_2 + (1-\theta_d)I_2} \\ \dfrac{\theta_e(1-\theta_d)\gamma\alpha S_2 I_2}{S_2 + (1-\theta_d)I_2} \\ \dfrac{\beta S_2 I_2}{S_2 + I_2} + \dfrac{(1-\theta_d)(1-\theta_e)\gamma\alpha S_1 I_1}{S_1 + (1-\theta_d)I_1} \\ \dfrac{\theta_e(1-\theta_d)\gamma\alpha S_1 I_1}{S_1 + (1-\theta_d)I_1} \end{pmatrix}$$

$$y = \begin{pmatrix} (c+d+\alpha)I_1 - (1-\theta_e)(1-\theta_d)\alpha I_2 \\ -\theta_d\alpha I_1 - \theta_e(1-\theta_d)\alpha I_2 + (e+f)Q_1 \\ (c+d+\alpha)I_2 - (1-\theta_e)(1-\theta_d)\alpha I_1 \\ -\theta_d\alpha I_2 - \theta_e(1-\theta_d)\alpha I_1 + (e+f)Q_2 \end{pmatrix}.$$

Kedua matriks tersebut akan dibentuk matriks Jacobinya, yaitu

$$K = \begin{pmatrix} \dfrac{\partial f_1(E_0)}{\partial I_1} & \dfrac{\partial f_1(E_0)}{\partial Q_1} & \dfrac{\partial f_1(E_0)}{\partial I_2} & \dfrac{\partial f_1(E_0)}{\partial Q_2} \\ \dfrac{\partial f_2(E_0)}{\partial I_1} & \dfrac{\partial f_2(E_0)}{\partial Q_1} & \dfrac{\partial f_2(E_0)}{\partial I_2} & \dfrac{\partial f_2(E_0)}{\partial Q_2} \\ \dfrac{\partial f_3(E_0)}{\partial I_1} & \dfrac{\partial f_3(E_0)}{\partial Q_1} & \dfrac{\partial f_3(E_0)}{\partial I_2} & \dfrac{\partial f_3(E_0)}{\partial Q_2} \\ \dfrac{\partial f_4(E_0)}{\partial I_1} & \dfrac{\partial f_4(E_0)}{\partial Q_1} & \dfrac{\partial f_4(E_0)}{\partial I_2} & \dfrac{\partial f_4(E_0)}{\partial Q_2} \end{pmatrix}$$

$$Y = \begin{pmatrix} \dfrac{\partial y_1(E_0)}{\partial I_1} & \dfrac{\partial y_1(E_0)}{\partial Q_1} & \dfrac{\partial y_1(E_0)}{\partial I_2} & \dfrac{\partial y_1(E_0)}{\partial Q_2} \\ \dfrac{\partial y_2(E_0)}{\partial I_1} & \dfrac{\partial y_2(E_0)}{\partial Q_1} & \dfrac{\partial y_2(E_0)}{\partial I_2} & \dfrac{\partial y_2(E_0)}{\partial Q_2} \\ \dfrac{\partial y_3(E_0)}{\partial I_1} & \dfrac{\partial y_3(E_0)}{\partial Q_1} & \dfrac{\partial y_3(E_0)}{\partial I_2} & \dfrac{\partial y_3(E_0)}{\partial Q_2} \\ \dfrac{\partial y_4(E_0)}{\partial I_1} & \dfrac{\partial y_4(E_0)}{\partial Q_1} & \dfrac{\partial y_4(E_0)}{\partial I_2} & \dfrac{\partial y_4(E_0)}{\partial Q_2} \end{pmatrix}$$

sehingga didapatkan

$$K=\begin{pmatrix}\beta & 0 & (1-\theta_d)(1-\theta_e)\gamma\alpha & 0\\ 0 & 0 & \theta_e(1-\theta_d)\gamma\alpha & 0\\ (1-\theta_d)(1-\theta_e)\gamma\alpha & 0 & \beta & 0\\ \theta_e(1-\theta_d)\gamma\alpha & 0 & 0 & 0\end{pmatrix}$$

$$Y=\begin{pmatrix}c+d+\alpha & 0 & -(1-\theta_e)(1-\theta_d)\alpha & 0\\ -\theta_d\alpha & e+f & -\theta_e(1-\theta_d)\alpha & 0\\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & c+d+\alpha & 0\\ -\theta_e(1-\theta_d)\alpha & 0 & -\theta_d\alpha & e+f\end{pmatrix}.$$

Invers dari matriks Y

$$\det(Y)=(e+f)\begin{vmatrix}c+d+\alpha & 0 & -(1-\theta_e)(1-\theta_d)\alpha\\ -\theta_d\alpha & e+f & -\theta_e(1-\theta_d)\\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & c+d+\alpha\end{vmatrix}$$

$$\det(Y)=(e+f)(e+f)\begin{vmatrix}c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha\\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha\end{vmatrix}$$

$$\det(Y)=(e+f)^2((c+d+\alpha)(c+d+\alpha)\\ -(-(1-\theta_e)(1-\theta_d)\alpha)(-(1-\theta_e)(1-\theta_d)\alpha))$$

$$\det(Y)=(e+f)^2((c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2)$$

Matriks adjoint Y

$$K_{11}=\begin{vmatrix}e+f & -\theta_e(1-\theta_d)\alpha & 0\\ 0 & c+d+\alpha & 0\\ 0 & -\theta_d\alpha & e+f\end{vmatrix}=(e+f)^2(c+d+\alpha)=\varrho$$

$$K_{12}=\begin{vmatrix}-\theta_d\alpha & -\theta_e(1-\theta_d)\alpha & 0\\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha & 0\\ -\theta_e(1-\theta_d)\alpha & -\theta_d\alpha & e+f\end{vmatrix}$$

$$=(e+f)\begin{vmatrix}-\theta_d\alpha & -\theta_e(1-\theta_d)\alpha\\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha\end{vmatrix}$$

$$=(e+f)(-\theta_d\alpha(c+d+\alpha)-\theta_e(1-\theta_e)(1-\theta_d)^2\alpha^2)=\tau$$

$$K_{13} = \begin{vmatrix} -\theta_d\alpha & e+f & 0 \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & 0 \\ -\theta_e(1-\theta_d)\alpha & 0 & e+f \end{vmatrix}$$
$$= (e+f)(0-(-(1-\theta_e)(1-\theta_d)\alpha)(e+f))$$
$$= (1-\theta_e)(1-\theta_d)\alpha)(e+f)^2 = \varsigma$$

$$K_{14} = \begin{vmatrix} -\theta_d\alpha & e+f & -\theta_e(1-\theta_d)\alpha \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & c+d+\alpha \\ -\theta_e(1-\theta_d)\alpha & 0 & -\theta_d\alpha \end{vmatrix}$$
$$= (e+f)\begin{vmatrix} -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha \\ -\theta_e(1-\theta_d)\alpha & -\theta_d\alpha \end{vmatrix}$$
$$= (e+f)(\theta_d(1-\theta_e)(1-\theta_d)\alpha^2 + (1-\theta_d)(c+d+\alpha)\theta_e\alpha) = \varepsilon$$

$$K_{21} = \begin{vmatrix} 0 & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ 0 & c+d+\alpha & 0 \\ 0 & -\theta_d\alpha & e+f \end{vmatrix} = 0$$

$$K_{22} = \begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha & 0 \\ -\theta_e(1-\theta_d)\alpha & -\theta_d\alpha & e+f \end{vmatrix}$$
$$= (e+f)\begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha \\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha \end{vmatrix}$$
$$= (e+f)((c+d+\alpha)^2 - (1-\theta_e)^2(1-\theta_d)^2\alpha^2) = \upsilon$$

$$K_{23} = \begin{vmatrix} c+d+\alpha & 0 & 0 \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & 0 \\ -\theta_e(1-\theta_d)\alpha & 0 & e+f \end{vmatrix} = 0$$

$$K_{24} = \begin{vmatrix} c+d+\alpha & 0 & -(1-\theta_e)(1-\theta_d)\alpha \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & c+d+\alpha \\ -\theta_e(1-\theta_d)\alpha & 0 & -\theta_d\alpha \end{vmatrix} = 0$$

$$K_{31} = \begin{vmatrix} 0 & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ e+f & -\theta_e(1-\theta_d)\alpha & 0 \\ 0 & -\theta_d\alpha & e+f \end{vmatrix}$$
$$= (e+f)(0-(e+f)(-(1-\theta_e)(1-\theta_d)\alpha))$$

$$= (e+f)^2(1-\theta_e)(1-\theta_d)\alpha = \varsigma$$

$$K_{32} = \begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ -\theta_d\alpha & -\theta_e(1-\theta_d)\alpha & 0 \\ -\theta_e(1-\theta_d)\alpha & -\theta_d\alpha & e+f \end{vmatrix}$$

$$= (e+f)\begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha \\ -\theta_d\alpha & -\theta_e(1-\theta_d)\alpha \end{vmatrix}$$

$$= (e+f)((c+d+\alpha)(-\theta_e(1-\theta_d)\alpha)$$
$$-(-\theta_d\alpha)(-(1-\theta_e)(1-\theta_d)\alpha))$$
$$= (e+f)(-(c+d+\alpha)(1-\theta_d)\theta_e\alpha - (1-\theta_e)(1-\theta_d)\theta_d\alpha^2) = \varsigma$$

$$K_{33} = \begin{vmatrix} c+d+\alpha & 0 & 0 \\ -\theta_d\alpha & e+f & 0 \\ -\theta_e(1-\theta_d)\alpha & 0 & e+f \end{vmatrix}$$

$$= (e+f)\begin{vmatrix} c+d+\alpha & 0 \\ -\theta_d\alpha & e+f \end{vmatrix}$$

$$= (e+f)^2(c+d+\alpha) = \varrho$$

$$K_{34} = \begin{vmatrix} c+d+\alpha & 0 & -(1-\theta_e)(1-\theta_d)\alpha \\ -\theta_d\alpha & e+f & -\theta_e(1-\theta_d)\alpha \\ -\theta_e(1-\theta_d)\alpha & 0 & -\theta_d\alpha \end{vmatrix}$$

$$= (e+f)\begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha \\ -\theta_e(1-\theta_d)\alpha & -\theta_d\alpha \end{vmatrix}$$

$$= (e+f)((c+d+\alpha)(-\theta_d\alpha)$$
$$-(-(1-\theta_e)(1-\theta_d)\alpha)(-\theta_e(1-\theta_d)\alpha))$$
$$= (e+f)(-(c+d+\alpha)\theta_d\alpha - (1-\theta_e)(1-\theta_d)^2\alpha^2\theta_e) = \tau$$

$$K_{41} = \begin{vmatrix} 0 & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ e+f & -\theta_e(1-\theta_d)\alpha & 0 \\ 0 & c+d+\alpha & 0 \end{vmatrix} = 0$$

$$K_{42} = \begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha & 0 \\ -\theta_d\alpha & -\theta_e(1-\theta_d)\alpha & 0 \\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha & 0 \end{vmatrix} = 0$$

$$K_{43} = \begin{vmatrix} c+d+\alpha & 0 & 0 \\ -\theta_d\alpha & e+f & 0 \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & 0 \end{vmatrix} = 0$$

$$K_{44} = \begin{vmatrix} c+d+\alpha & 0 & -(1-\theta_e)(1-\theta_d)\alpha \\ -\theta_d\alpha & e+f & -\theta_e(1-\theta_d)\alpha \\ -(1-\theta_e)(1-\theta_d)\alpha & 0 & c+d+\alpha \end{vmatrix}$$
$$= (e+f)\begin{vmatrix} c+d+\alpha & -(1-\theta_e)(1-\theta_d)\alpha \\ -(1-\theta_e)(1-\theta_d)\alpha & c+d+\alpha \end{vmatrix}$$
$$= (e+f)((c+d+\alpha)^2 - (1-\theta_e)^2(1-\theta_d)^2\alpha^2) = \upsilon$$

$$K = \begin{pmatrix} \varrho & \tau & \varsigma & \varepsilon \\ 0 & \upsilon & 0 & 0 \\ \varsigma & \chi & \varrho & \tau \\ 0 & 0 & 0 & \upsilon \end{pmatrix}$$

$$K^T = \begin{pmatrix} \varrho & 0 & \varsigma & 0 \\ \tau & \upsilon & \chi & 0 \\ \varsigma & 0 & \varrho & 0 \\ \varepsilon & 0 & \tau & \upsilon \end{pmatrix}$$

Adj $(Y) = K^T$

Jadi $Y^{-1} = \frac{1}{\det(Y)}\ \mathrm{adj}(Y)$

$$\det(Y) = (e+f)^2((c+d+\alpha)^2 - (1-\theta_e)^2(1-\theta_d)^2\alpha^2) = (e+f)\upsilon$$

$$Y^{-1} = \frac{1}{(e+f)\upsilon}\begin{pmatrix} \varrho & 0 & \varsigma & 0 \\ \tau & \upsilon & \chi & 0 \\ \varsigma & 0 & \varrho & 0 \\ \varepsilon & 0 & \tau & \upsilon \end{pmatrix}$$

$$Y^{-1} = \begin{pmatrix} \frac{\varrho}{(e+f)\upsilon} & 0 & \frac{\varsigma}{(e+f)\upsilon} & 0 \\ \frac{\tau}{(e+f)\upsilon} & \frac{\upsilon}{(e+f)\upsilon} & \frac{\chi}{(e+f)\upsilon} & 0 \\ \frac{\varsigma}{(e+f)\upsilon} & 0 & \frac{\varrho}{(e+f)\upsilon} & 0 \\ \frac{\varepsilon}{(e+f)\upsilon} & 0 & \frac{\tau}{(e+f)\upsilon} & \frac{1}{(e+f)} \end{pmatrix}$$

sehingga Matriks Generasi Selanjutnya yaitu

$$FY^{-1} = \begin{pmatrix} \beta & 0 & (1-\theta_d)(1-\theta_e)\gamma\alpha & 0 \\ 0 & 0 & \theta_e(1-\theta_d)\gamma\alpha & 0 \\ (1-\theta_d)(1-\theta_e)\gamma\alpha & 0 & \beta & 0 \\ \theta_e(1-\theta_d)\gamma\alpha & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{\varrho}{(e+f)\upsilon} & 0 & \frac{\varsigma}{(e+f)\upsilon} & 0 \\ \frac{\tau}{(e+f)\upsilon} & \frac{\upsilon}{(e+f)\upsilon} & \frac{\chi}{(e+f)\upsilon} & 0 \\ \frac{\varsigma}{(e+f)\upsilon} & 0 & \frac{\varrho}{(e+f)\upsilon} & 0 \\ \frac{\varepsilon}{(e+f)\upsilon} & 0 & \frac{\tau}{(e+f)\upsilon} & \frac{1}{(e+f)} \end{pmatrix}$$

$$FY^{-1} = \begin{pmatrix} \frac{\beta\varrho + (1-\theta_d)(1-\theta_e)\gamma\alpha\varsigma}{(e+f)\upsilon} & 0 & \frac{\beta\varsigma + (1-\theta_d)(1-\theta_e)\gamma\alpha\varrho}{(e+f)\upsilon} & 0 \\ \frac{\varsigma\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} & 0 & \frac{\varrho\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} & 0 \\ \frac{\varrho(1-\theta_d)(1-\theta_e)\gamma\alpha + \varsigma\beta}{(e+f)\upsilon} & 0 & \frac{\varsigma(1-\theta_d)(1-\theta_e)\gamma\alpha + \varrho\beta}{(e+f)\upsilon} & 0 \\ \frac{\varrho\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} & 0 & \frac{\varsigma\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} & \frac{1}{(e+f)} \end{pmatrix}$$

dimisalkan

$$\delta = \frac{\beta\varrho + (1-\theta_d)(1-\theta_e)\gamma\alpha\varsigma}{(e+f)\upsilon}$$

$$= \frac{\beta(c+d+\alpha) + (1-\theta_e)^2(1-\theta_d)^2\alpha^2\gamma}{(c+d+\alpha)^2 - (1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\varepsilon = \frac{\varsigma\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} = \frac{(1-\theta_e)^2(1-\theta_d)^2\theta_e\alpha^2\gamma}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\omega = \frac{\varrho(1-\theta_d)(1-\theta_e)\gamma\alpha + \varsigma\beta}{(e+f)\upsilon}$$

$$= \frac{\beta(1-\theta_e)(1-\theta_d)\alpha + (1-\theta_e)(1-\theta_d)\alpha\gamma(c+d+\alpha)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\varphi = \frac{\varrho\theta_e(1-\theta_d)\gamma\alpha}{(e+f)\upsilon} = \frac{(c+d+\alpha)(1-\theta_d)\theta_e\alpha\gamma}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$KY^{-1} = \begin{pmatrix} \delta & 0 & \varphi & 0 \\ \varepsilon & 0 & \varphi & 0 \\ \omega & 0 & \delta & 0 \\ \varphi & 0 & \varepsilon & 0 \end{pmatrix}$$

Kemudian dicari nilai eigen dari matriks KY^{-1} yaitu,

$$|KY^{-1} - I\lambda| = 0$$

$$\begin{vmatrix} \delta-\lambda & 0 & \omega & 0 \\ \varepsilon & -\lambda & \varphi & 0 \\ \omega & 0 & \delta-\lambda & 0 \\ \varphi & 0 & \varepsilon & -\lambda \end{vmatrix} = 0$$

$$(-\lambda)\begin{vmatrix} \delta-\lambda & 0 & \omega \\ \varepsilon & -\lambda & \varphi \\ \omega & 0 & \delta-\lambda \end{vmatrix} = 0$$

$$(-\lambda)(-\lambda)\begin{vmatrix} \delta-\lambda & \omega \\ \omega & \delta-\lambda \end{vmatrix} = 0$$

$$\lambda^2((\delta-\lambda)^2-\omega^2) = 0$$

$$\lambda^2\big((\delta-\lambda)-\omega\big)((\delta-\lambda)+\omega) = 0$$

$$\lambda^2\big((\delta-\omega)-\lambda\big)((\delta+\omega)-\lambda)) = 0$$

$$\lambda_{1,2} = 0$$

$$\lambda_3 = \delta-\omega$$

$$\lambda_4 = \delta+\omega$$

Spectral radius dari dari matriks KY^{-1} adalah $\delta + \omega$

$$\delta+\omega = \frac{\beta(c+d+\alpha)+(1-\theta_e)^2(1-\theta_d)^2\alpha^2\gamma}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2} + \frac{\beta(1-\theta_e)(1-\theta_d)\alpha+(1-\theta_e)(1-\theta_d)\alpha\gamma(c+d+\alpha)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\delta+\omega = \frac{\beta(c+d+\alpha)+(1-\theta_e)(1-\theta_d)\alpha\gamma(c+d+\alpha)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2} + \frac{\beta(1-\theta_e)(1-\theta_d)\alpha+(1-\theta_e)^2(1-\theta_d)^2\alpha^2\gamma}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\delta+\omega = \frac{(c+d+\alpha)(\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2} + \frac{(1-\theta_e)(1-\theta_d)\alpha(\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\delta+\omega = \frac{((c+d+\alpha)+(1-\theta_e)(1-\theta_d)\alpha)(\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma)}{(c+d+\alpha)^2-(1-\theta_e)^2(1-\theta_d)^2\alpha^2}$$

$$\delta+\omega = \frac{\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma}{(c+d+\alpha)-(1-\theta_e)(1-\theta_d)\alpha)}$$

$$\delta+\omega = \frac{\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma}{c+d+\theta_e\alpha+(1-\theta_e)\theta_d\alpha)}$$

sehingga

$$R_0 = \frac{\beta+(1-\theta_e)(1-\theta_d)\alpha\gamma}{c+d+\theta_e\alpha+(1-\theta_e)\theta_d\alpha)}$$

Lampiran 3 Perhitungan Lemma 2.1 untuk Matriks $A_1 + B_1$

1. $\boldsymbol{L_1 = -\mathrm{tr}(J) > 0}$

$L_1 = -\mathrm{tr}(J) > 0$ atau $\mathrm{tr}(J) < 0$
$\mathrm{tr}(J) = (a_{11} + a_{22} + a_{33})$

$$a_{11} = -b - \frac{\beta I^{*2}}{(S^* + I^*)^2} - \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2} < 0$$

$$a_{22} = -(c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) + \frac{\beta S^{*2}}{(S^* + I^*)^2} + \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^* + (1-\theta_d)I^*)^2}$$

pada kompartemen infeksi model 3.1 dengan kondisi

$$\frac{\beta S}{S+I} > \frac{\beta S^2}{(S+I)^2} \text{ dan } \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha S}{S + (1-\theta_d)I} > \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^2}{(S + (1-\theta_d)I)^2}$$

$$\frac{\beta SI}{S+I} - (c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha)I - \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha SI}{S + (1-\theta_d)I} = 0$$

$$\frac{\beta S}{S+I} - (c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) - \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha S}{S + (1-\theta_d)I} = 0$$

$$(c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) = \frac{\beta S}{S+I} + \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha S}{S + (1-\theta_d)I}$$

$$a_{22} = -\left(\frac{\beta S^*}{S^* + I^*} + \frac{\gamma(1-\theta_e)(1-\theta_d)\alpha S^*}{S^* + (1-\theta_d)I^*}\right) + \frac{\beta S^{*2}}{(S^* + I^*)^2} + \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^* + (1-\theta_d)I^*)^2}$$

$a_{22} < 0$
$a_{33} = -(e + f) < 0$
sehingga terbukti $\mathrm{tr}(J) = (a_{11} + a_{22} + a_{33}) < 0$

2. $\boldsymbol{L_2 = J_1 + J_2 + J_3 > 0}$

$J_3 = a_{22}a_{33} > 0$ karena $a_{22} < 0$ dan $a_{33} < 0$

$$J_2 = a_{11}a_{33} - a_{31}a_{13}$$

$$J_2 = (e+f)\left(b + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right) - f\frac{\theta_e(1-\theta_d)^2\gamma\alpha I^2}{(S^* + (1-\theta_d)I^*)^2}$$

$$J_2 = e\left(b + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right) + f\left(b + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2} - \frac{\theta_e(1-\theta_d)^2\gamma\alpha I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right)$$

$$J_2 = e\left(b + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right) + f\left(b + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{\gamma\alpha(1-\theta_e)(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right)$$

sehingga terbukti $J_2 > 0$

$$J_1 = a_{11}a_{22} - a_{21}a_{12} = (a_{11} + a_{21})a_{22} - a_{21}(a_{12} + a_{22})$$

$$(a_{11} + a_{21})a_{22} - a_{21}(a_{12} + a_{22})$$
$$= \left(-b - \frac{\beta I^{*2}}{(S^* + I^*)^2} - \frac{\gamma\alpha(1-\theta_d)^2 I^{*2}}{(S^* + (1-\theta_d)I^*)^2} + \frac{\beta I^{*2}}{(S^* + I^*)^2} + \frac{(1-\theta_e)(1-\theta_d)^2\gamma\alpha I^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right)(a_{22}) - a_{21}\left(d - \frac{\beta S^{*2}}{(S^* + I^*)^2} - \frac{\gamma\alpha(1-\theta_d)S^{*2}}{(S^* + (1-\theta_d)I^*)^2} - (c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) + \frac{\beta S^{*2}}{(S^* + I^*)^2} + \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^* + (1-\theta_d)I^*)^2}\right)$$

$$= \left(-b - \frac{\theta_e(1-\theta_d)^2\gamma\alpha I^{*2}}{(S^* + (1-\theta_d)I^*)^2} \right)(a_{22}) - a_{21}\left(-(c + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) - \frac{\theta_e(1-\theta_d)\gamma\alpha S^{*2}}{(S^* + (1-\theta_d)I^*)^2} \right)$$

$$= \left(b + \frac{\theta_e(1-\theta_d)^2\gamma\alpha I^{*2}}{(S^* + (1-\theta_d)I^*)^2} \right)(-a_{22}) + a_{21}\left((c + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) + \frac{\theta_e(1-\theta_d)\gamma\alpha S^{*2}}{(S^* + (1-\theta_d)I^*)^2} \right)$$

Karena $-a_{22} > 0$ dan $a_{21} > 0$ terbukti bahwa $J_1 > 0$

Terbukti juga $L_2 = J_1 + J_2 + J_3 > 0$

3. **$L_3 = -\det(J)$**

$$\det(J) = -eJ_1 - fa_{22}(a_{11} + a_{21} + a_{31}) + fa_{21}(a_{12} + a_{22} + a_{32})$$
$$= -eJ_1 - fa_{22}(-b) + fa_{21}(-c)$$
$$= -eJ_1 - fb(-a_{22}) - fa_{21}(c)$$

Karena $J_1 > 0$, $-a_{22} > 0$ dan $a_{21} > 0$ terbukti $\det(J) < 0$
sehingga $L_3 = -\det(J) > 0$

4. **$L_1L_2 - L_3$**

$$L_1L_2 - L_3 = -a_{11}(J_1 + J_2 + J_3) - a_{22}(J_1 + J_3) - a_{33}(J_2 + J_3) - a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32}$$

$-a_{11}(J_1 + J_2 + J_3) > 0$ karena $a_{11} < 0$ dan $J_1 + J_2 + J_3 > 0$
$-a_{22}(J_1 + J_3) > 0$ karena $a_{22} < 0$ dan $J_1 + J_3 > 0$
$-a_{33}(J_2 + J_3) > 0$ karena $a_{33} < 0$ dan $J_2 + J_3 > 0$
$-a_{11}a_{22}a_{33} > 0$ karena $a_{11}, a_{22}, a_{33} < 0$
$a_{13}a_{21}a_{32} > 0$ karena $a_{13}, a_{21}, a_{32} > 0$
sehingga terbukti bahwa $L_1L_2 - L_3 > 0$

Lampiran 4 Perhitungan Lemma 2.1 untuk Matriks $A_1 - B_1$

Pada perhitungan ini dibagi menjadi dua kasus yaitu $a_{21} > 0$ dan $a_{21} \leq 0$.

Kasus $\boldsymbol{a_{21} > 0}$

1. $\boldsymbol{L_1 = -\mathrm{tr}(J) > 0}$ **atau** $\boldsymbol{\mathrm{tr}(J) < 0}$

$a_{11} = -b - 2\alpha - \psi_3 I^{*2} + \psi_1 < -b - (2-\gamma)\alpha - \psi_3 I^{*2} < 0$

$$a_{22} = -(c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) + \psi_3 S^{*2} - (1-\theta_e)\psi_2$$
$$= -\left(\psi_3 S^* + \frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*}\right) + \psi_3 S^{*2} - (1-\theta_e)\psi_2$$
$$= -\left(\frac{\beta S^*}{S^*+I^*} + \frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*}\right) + \frac{\beta S^{*2}}{(S^*+I^*)^2} - \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2}$$

karena $\frac{\beta S^*}{S^*+I^*} > \frac{\beta S^{*2}}{(S^*+I^*)^2}$ dan $\frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*} > \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2}$

maka $a_{22} < 0$

$a_{33} = -(e+f) < 0$

Sehingga terbukti $\mathrm{tr}(J) = (a_{11} + a_{22} + a_{33}) < 0$

2. $\boldsymbol{L_2 = J_1 + J_2 + J_3 > 0}$

$J_3 = a_{22}a_{33}$, karena $a_{22} < 0$ dan $a_{33} < 0$ maka $J_3 > 0$

$J_2 = a_{11}a_{33} - a_{31}a_{13}$, karena $a_{11} < 0, a_{33} < 0$ dan $a_{13} < 0$ maka $J_2 > 0$

$J_1 = a_{11}a_{22} - a_{21}a_{12} = (a_{11} + a_{21})a_{22} - a_{21}(a_{12} + a_{22})$, karena $a_{22} < 0, (a_{11} + a_{21}) < 0$ dan $a_{12} + a_{22} < 0$ maka $J_1 > 0$

Terbukti juga $L_2 = J_1 + J_2 + J_3 > 0$

3. $\boldsymbol{L_3 = -\det(J) < 0}$ **atau** $\boldsymbol{\det(J) > 0}$

$$\det(J) = -eJ_1 - fa_{22}(a_{11} + a_{21} + a_{31}) + fa_{21}(a_{12} + a_{22} + a_{32})$$
$$= -eJ_1 - fa_{22}(-b - 2\alpha) + fa_{21}(-2\psi_2 - 2\alpha + 2\theta_d\alpha - c)$$
$$= -eJ_1 - fb(-a_{22}) - fa_{21}(-2\psi_2 - (1-\theta_d)2\alpha - c)$$
$$= -\left(eJ_1 + fb(a_{22}) + fa_{21}(2\psi_2 + (1-\theta_d)2\alpha + c)\right) < 0$$

Terbukti bahwa $\det(J) > 0$

4. $\boldsymbol{L_1L_2 - L_3 > 0}$

$$L_1L_2 - L_3 = -a_{11}(J_1 + J_2 + J_3) - a_{22}(J_1 + J_3) - a_{33}(J_2 + J_3) - a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32}$$

$-a_{11}(J_1 + J_2 + J_3) > 0$ karena $a_{11} < 0$ dan $J_1 + J_2 + J_3 > 0$
$-a_{22}(J_1 + J_3) > 0$ karena $a_{22} < 0$ dan $J_1 + J_3 > 0$
$-a_{33}(J_2 + J_3) > 0$ karena $a_{33} < 0$ dan $J_2 + J_3 > 0$
$-a_{11}a_{22}a_{33} > 0$ karena $a_{11}, a_{22}, a_{33} < 0$
$a_{13}a_{21}a_{32} > 0$ karena $a_{13}, a_{21}, a_{32} > 0$
Sehingga terbukti bahwa $L_1L_2 - L_3 > 0$

Kasus $\boldsymbol{a_{21} \leq 0}$

1. $\boldsymbol{L_1 = -\mathrm{tr}(J) > 0}$ **atau** $\boldsymbol{\mathrm{tr}(J) < 0}$

$$a_{11} = -b - 2\alpha - \psi_3 I^{*2} + \psi_1 < -b - (2-\gamma)\alpha - \psi_3 I^{*2} < 0$$

$$\begin{aligned} a_{22} &= -(c + d + \theta_e\alpha + (1-\theta_e)\theta_d\alpha) + \psi_3 S^{*2} - (1-\theta_e)\psi_2 \\ &= -\left(\psi_3 S^* + \frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*}\right) + \psi_3 S^{*2} - (1-\theta_e)\psi_2 \\ &= -\left(\frac{\beta S^*}{S^*+I^*} + \frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*}\right) + \frac{\beta S^{*2}}{(S^*+I^*)^2} - \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2} \end{aligned}$$

karena $\frac{\beta S^*}{S^*+I^*} > \frac{\beta S^{*2}}{(S^*+I^*)^2}$ dan $\frac{(1-\theta_d)\gamma\alpha S^*}{S^*+(1-\theta_d)I^*} > \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2}$
maka $a_{22} < 0$

$a_{33} = -(e + f) < 0$
Sehingga terbukti $\mathrm{tr}(J) = (a_{11} + a_{22} + a_{33}) < 0$

2. $\boldsymbol{L_2 = J_1 + J_2 + J_3 > 0}$

$J_3 = a_{22}a_{33}$, karena $a_{22} < 0$ dan $a_{33} < 0$ maka $J_3 > 0$
$J_2 = a_{11}a_{33} - a_{31}a_{13}$, karena $a_{11} < 0, a_{33} < 0$ dan $a_{13} < 0$ maka $J_2 > 0$

$J_1 = a_{11}a_{22} - a_{21}a_{12}$,
karena $a_{21} \leq 0$ atau $a_{21} = \frac{\beta S^{*2}}{(S^*+I^*)^2} - \frac{(1-\theta_e)(1-\theta_d)^2\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2} \leq 0.$

$$\frac{\beta}{(S^*+I^*)^2} \leq \frac{(1-\theta_e)(1-\theta_d)^2\gamma\alpha}{(S^*+(1-\theta_d)I^*)^2} < \frac{(1-\theta_e)(1-\theta_d)\gamma\alpha}{(S^*+(1-\theta_d)I^*)^2} < \frac{(1-\theta_d)\gamma\alpha}{(S^*+(1-\theta_d)I^*)^2}.$$

sehingga nilai $a_{12} = d - \frac{\beta S^{*2}}{(S^*+I^*)^2} + \frac{(1-\theta_d)\gamma\alpha S^{*2}}{(S^*+(1-\theta_d)I^*)^2} > 0$
karena $\frac{\beta}{(S^*+I^*)^2} < \frac{(1-\theta_d)\gamma\alpha}{(S^*+(1-\theta_d)I^*)^2}$

$a_{21} \leq 0, a_{12} > 0$ dan $a_{11}a_{22} > 0$ maka $J_1 > 0$
Terbukti juga $L_2 = J_1 + J_2 + J_3 > 0$

3. $\boldsymbol{L_3 = -\det(J) < 0}$ **atau** $\boldsymbol{\det(J) > 0}$

$$\det(J) = eJ_1 - fa_{22}(a_{11} + a_{21} + a_{31}) + fa_{21}(a_{12} + a_{22} + a_{32})$$

$$= -eJ_1 - f[(-a_{22})(b + 2\alpha) - a_{21}(c + 2\alpha - 2\theta_d\alpha)]$$

$$= -eJ_1 - f[(-a_{22})(b + 2\alpha) - a_{21}(c + 2(1 - \theta_d)\alpha)]$$

$$= -\left[eJ_1 + f[(-a_{22})(b + 2\alpha) - a_{21}(c + 2(1 - \theta_d)\alpha)]\right]$$

Karena $a_{22} < 0$ dan $a_{21} \leq 0$ sehingga

$$= -\left[eJ_1 + f[(-a_{22})(b + 2\alpha) - a_{21}(c + 2(1 - \theta_d)\alpha)]\right] < 0$$

Terbukti bahwa $-\det(J) < 0$

4. $\boldsymbol{L_1L_2 - L_3}$

$$L_1L_2 - L_3 = -a_{11}(J_1 + J_2 + J_3) - a_{22}(J_1 + J_3) - a_{33}(J_2 + J_3) - a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32}$$

$-a_{11}(J_1 + J_2 + J_3) > 0$ karena $a_{11} < 0$ dan $J_1 + J_2 + J_3 > 0$
$-a_{22}(J_1 + J_3) > 0$ karena $a_{22} < 0$ dan $J_1 + J_3 > 0$
$-a_{33}(J_2 + J_3) > 0$ karena $a_{33} < 0$ dan $J_2 + J_3 > 0$
$-a_{11}a_{22}a_{33} > 0$ karena $a_{11}, a_{22}, a_{33} < 0$
$a_{13}a_{21}a_{32} > 0$ karena $a_{13} > 0$ dan $a_{21} \leq 0, a_{32} < 0$
Sehingga terbukti bahwa $L_1L_2 - L_3 > 0$

Lampiran 5 *Listing* Program RK4 Pada Simulasi Model

```
function dy=SIQS(t,y);
%nilai parameter :
a=2;
b=0.2;
c=0.5;
d=0.5;
e=0.4;
f=1.2;
vd=0.1;
ve=0.8;
gamma=0.9;
alpha=0.1;
beta=0.7;
R0=(beta+(1-ve)*(1-
vd)*gamma*alpha)/(c+d+ve*alpha+(1-ve)*vd*alpha);

%model SIQS :
dy(1)=a-((beta*y(1)*y(2))/(y(1)+y(2)))-
b*y(1)+d*y(2)+f*y(3)-alpha*y(1)+alpha*y(4)-
((gamma*(1-vd)*alpha*y(4)*y(5))/(y(4)+(1-
vd)*y(5)));
dy(4)=a-((beta*y(4)*y(5))/(y(4)+y(5)))-
b*y(4)+d*y(5)+f*y(6)-alpha*y(4)+alpha*y(1)-
((gamma*(1-vd)*alpha*y(1)*y(2))/(y(1)+(1-
vd)*y(2)));

dy(2)=((beta*y(1)*y(2))/(y(1)+y(2)))-
(c+d+alpha)*y(2)+(1-ve)*(1-vd)*alpha*y(5)+((1-
ve)*(1-vd)*gamma*alpha*y(4)*y(5))/(y(4)+(1-
vd)*y(5));
dy(5)=((beta*y(4)*y(5))/(y(4)+y(5)))-
(c+d+alpha)*y(5)+(1-ve)*(1-vd)*alpha*y(2)+((1-
ve)*(1-vd)*gamma*alpha*y(1)*y(2))/(y(4)+(1-
vd)*y(2));

dy(3)=vd*alpha*y(2)+ve*(1-vd)*alpha*y(5)+(ve*(1-
vd)*gamma*alpha*y(4)*y(5))/(y(4)+(1-vd)*y(5))-
(e+f)*y(3);
dy(6)=vd*alpha*y(5)+ve*(1-vd)*alpha*y(2)+(ve*(1-
vd)*gamma*alpha*y(1)*y(2))/(y(1)+(1-vd)*y(2))-
```

```
(e+f)*y(6);
```

```
%Perhitungan Solusi Persamaan dengan Metode RK4
function [t,z]=RK4(SIQS,a,b,za,M)
h=(b-a)/M;
t=a:h:b;
z=zeros(M+1,length(za));
z(1,:)=za;
for i=1:M
    k1=h*feval(SIQS,t(i),z(i,:));
    k2=h*feval(SIQS,t(i)+h/2,z(i,:)+k1/2);
    k3=h*feval(SIQS,t(i)+h/2,z(i,:)+k2/2);
    k4=h*feval(SIQS,t(i)+h,z(i,:)+k3);
    z(i+1,:)=z(i,:)+(k1+2*k2+2*k3+k4)/6;
end;
```

```
%Program simulasi utama
clear all;
clc
[t y]=RK4('SIQS',0,10000,[0.2 0.9 0.5 1 0
0],40000);
%[t y]=RK4('SIQS',0,10000,[8 8 3 9 2 8],40000);
%[t y]=RK4('SIQS',0,1000,[0.2 0.4 6 0 2 13],4000);
%[t y]=RK4('SIQS',0,1000,[0 2 3 1 2 0],4000);

a=2;
b=0.2;
c=0.5;
d=0.5;
e=0.4;
f=1.2;
vd=0.1;
ve=0.3;
gamma=0.9;
alpha=0.1;
beta=0.9803;

R0=(beta+(1-ve)*(1-
vd)*gamma*alpha)/(c+d+ve*alpha+(1-ve)*vd*alpha)
pi=c+d+ve*alpha+(1-ve)*vd*alpha;
shi=pi*(R0-1);
bebe=(shi-pi*(1-vd)-beta*vd);
akar=sqrt(bebe^2+4*shi*pi*(1-vd))
m=(-bebe+akar)/2*shi;
```

```
n=(vd*alpha+ve*(1-vd)*alpha+(ve*(1-
vd)*gamma*alpha*m)/(m+(1-vd)))/(e+f);

S=m*a/(b*m+c+e*n)
I=a/(b*m+c+e*n)
Q=n*a/(b*m+c+e*n)

figure(1);
plot3(y(:,1),y(:,2),y(:,3),'LineWidth',2);
hold on
plot3(y(1,1),y(1,2),y(1,3),'*g','LineWidth',5);
%plot3(y(1000,1),y(1000,2),y(1000,3),'*b','LineWid
th',5);
plot3(a/b,0,0,'*magenta','LineWidth',5);
plot3(S,I,Q,'*r','LineWidth',5);
hold off
hold on
xlabel('Susceptible');
ylabel('infective');
zlabel('Quarantine');
legend('Potret fase','Nilai awal
(S(0),I(0),Q(0))','Titik bebas penyakit
(S,I,Q)','Titik endemi (S,I,Q)');
grid on
figure(2);
plot3(y(:,4),y(:,5),y(:,6),'LineWidth',2);
hold on
plot3(y(1,4),y(1,5),y(1,6),'*g','LineWidth',5);
%plot3(y(1000,4),y(1000,5),y(1000,6),'*b','LineWid
th',5);
plot3(a/b,0,0,'*magenta','LineWidth',5);
plot3(S,I,Q,'*r','LineWidth',5);
hold off
hold on
xlabel('Susceptible');
ylabel('infective');
zlabel('Quarantine');
legend('Potret fase','Nilai awal
(S(0),I(0),Q(0))','Titik bebas penyakit
(S,I,Q)','Titik endemi (S,I,Q)');
grid on
```

Lampiran 6 *Listing* Program pengaruh *exit-entry screening*

```
%Pengaruh perubahan teta d atau pemeriksaan
keluar
clc
clear all;
h=0.1;
N=1/h;

a=2;
b=0.2;
c=0.5;
d=0.5;
e=0.4;
f=1.2;
vdd=0;
ve=0.1;
gamma=0.9;
alpha=0.1;
beta=0.7;

for i=1:N+1
vd(i)=vdd+(i-1)*h;
R01=((1-ve)*(1-
vd(i)*gamma*alpha)/(c+d+ve*alpha+(1-
ve)*vd(i)*alpha));
R02=beta/(c+d+ve*alpha+(1-ve)*vd(i)*alpha);
R0=R01+R02;

pi=c+d+ve*alpha+(1-ve*vd(i))*alpha;
shi=pi*(R0-1);

bebe=(shi-pi*(1-vd(i))-beta*vd(i));
akar=sqrt(bebe^2+4*shi*pi*(1-vd(i)));

m=(-bebe+akar)/2*shi;
n=(vd(i)*alpha+ve*(1-vd(i))*alpha+(ve*(1-
vd(i))*gamma*alpha*m)/(m+(1-vd(i))))/(e+f);

S(i)=m*a/(b*m+c+e*n);
I(i)=a/(b*m+c+e*n);
```

```
Q(i)=n*a/(b*m+c+e*n);
end
figure(1)
plot(vd,S,'g',vd,I,'r',vd,Q,'b');
xlabel('Theta d');
ylabel('S*, I*, Q*');
axis square
legend('Susceptible','Infective','Quarantine');
```

```
%Pengaruh perubahan teta e atau pemeriksaan
masuk
clc
clear all;
h=0.1;
N=1/h;

a=2;
b=0.2;
c=0.5;
d=0.5;
e=0.4;
f=1.2;
vd=0.1;
vee=0;
gamma=0.9;
alpha=0.1;
beta=2;



for i=1:N+1
ve(i)=vee+(i-1)*h;
R01=((1-ve(i))*(1-
vd*gamma*alpha)/(c+d+ve(i)*alpha+(1-
ve(i))*vd*alpha));
R02=beta/(c+d+ve(i)*alpha+(1-ve(i))*vd*alpha);
R0=R01+R02;

pi=c+d+ve(i)*alpha+(1-ve(i)*vd)*alpha;
shi=pi*(R0-1);

bebe=(shi-pi*(1-vd)-beta*vd);
akar=sqrt(bebe^2+4*shi*pi*(1-vd));
```

```
m=(-bebe+akar)/2*shi;
n=(vd*alpha+ve(i)*(1-vd)*alpha+(ve(i)*(1-
vd)*gamma*alpha*m)/(m+(1-vd)))/(e+f);

S(i)=m*a/(b*m+c+e*n);
I(i)=a/(b*m+c+e*n);
Q(i)=n*a/(b*m+c+e*n);
end
figure(1)
plot(ve,S,'g',ve,I,'r',ve,Q,'b');
xlabel('Theta e');
ylabel('S*, I*, Q*');
axis square
legend('Susceptible','Infective','Quarantine');
```