

Lampiran 1 (Perhitungan Titik Keseimbangan)

$$\frac{dx}{dt} = r_1 x \left[1 - \frac{x+y}{K} \right] - \frac{bxy}{x+y+a} = 0$$

$$x \left[r_1 - \frac{r_1 x + r_1 y}{K} - \frac{by}{x+y+a} \right] = 0$$

$$x = 0 \text{ atau } r_1 - \frac{r_1 x + r_1 y}{K} - \frac{by}{x+y+a} = 0$$

$$\frac{dy}{dt} = r_2 y \left[1 - \frac{x+y}{K} \right] + \frac{bxy}{x+y+a} - \alpha y = 0$$

$$y \left[r_2 - \frac{r_2 x + r_2 y}{K} + \frac{bx}{x+y+a} - \alpha \right] = 0$$

$$y = 0 \text{ atau } r_2 - \frac{r_2 x + r_2 y}{K} + \frac{bx}{x+y+a} - \alpha = 0$$

Titik tetap ke-1

$$(x, y) = (0, 0)$$

Titik tetap ke-2

$$y = 0 \rightarrow r_1 - \frac{r_1 x}{K} = 0$$

$$Kr_1 - r_1 x = 0$$

$$Kr_1 = r_1 x$$

$$x = K$$

$$(x, y) = (K, 0)$$

Titik tetap ke-3

$$x = 0 \rightarrow r_2 - \frac{r_2 y}{K} - \alpha = 0$$

$$\frac{r_2 y}{K} = r_2 - \alpha$$

$$r_2 y = K(r_2 - \alpha)$$

$$y = \frac{K}{r_2}(r_2 - \alpha)$$

$$(x, y) = \left(0, \frac{K}{r_2}(r_2 - \alpha)\right) = (0, \bar{y})$$

Titik tetap ke-4

$$r_1 - \frac{r_1 x + r_1 y}{K} - \frac{by}{x + y + a} = 0$$

$$Kr_1(x + y + a) - (r_1 x + r_1 y)(x + y + a) - bKy = 0$$

$$Kr_1 x + Kr_1 y + Kr_1 a - (r_1 x^2 + r_1 xy + r_1 ax + r_1 xy + r_1 y^2 + r_1 ay) - bKy = 0$$

$$-r_1 x^2 + (Kr_1 - 2r_1 y - r_1 a)x + (Kr_1 y + Kr_1 a - r_1 y^2 - r_1 ay - bKy) = 0$$

$$r_1 x^2 + (-Kr_1 + 2r_1 y + r_1 a)x + (-Kr_1 y - Kr_1 a + r_1 y^2 + r_1 ay + bKy) = 0$$

$$x_{1,2}^* = \frac{-(-Kr_1 + 2r_1 y + r_1 a) \pm \sqrt{(-Kr_1 + 2r_1 y + r_1 a)^2 - 4r_1(-Kr_1 y - Kr_1 a + r_1 y^2 + r_1 ay + bKy)}}{2r_1}$$

$$x^* = \frac{(Kr_1 - 2r_1 y - r_1 a) + \sqrt{K^2 r_1^2 + 2Kr_1^2 a + r_1^2 a^2 - 4r_1 bKy}}{2r_1}$$

$$r_2 - \frac{r_2x + r_2y}{K} + \frac{bx}{x + y + a} - \alpha = 0$$

$$Kr_2(x + y + a) - (r_2x + r_2y)(x + y + a) + bKx - \alpha K(x + y + a) = 0$$

$$Kr_2x + Kr_2y + Kr_2a - (r_2x^2 + r_2xy + r_2ax + r_2xy + r_2y^2 + r_2ay) + bKx - (\alpha Kx + \alpha Ky + \alpha Ka) = 0$$

$$r_2y^2 + (-Kr_2 - 2r_2x + r_2a + \alpha K)y + (-Kr_2x - Kr_2a + r_2x^2 + r_2ax - bKx + \alpha Kx + \alpha Ka) = 0$$

$$r_1x^2 + (-Kr_1 + 2r_1y + r_1a)x + (-Kr_1y - Kr_1a + r_1y^2 + r_1ay + bKy) = 0$$

$$y_{1,2}^* = \frac{-(-Kr_2 + 2r_2x + r_2a + \alpha K) \pm \sqrt{(-Kr_2 + 2r_2x + r_2a + \alpha K)^2 - 4r_2(-Kr_2x - Kr_2a + r_2x^2 + r_2ax - bKx + \alpha Kx + \alpha Ka)}}{2r_2}$$

$$y^* = \frac{-M + \sqrt{M^2 - 4r_2N}}{2r_2}$$

$$M = (2xr_2 + ar_2 - K(r_2 - \alpha))$$

$$N = (r_2x^2 + x(r_2a - K(r_2 - \alpha) - Kb) - Ka(r_2 - \alpha))$$

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Lampiran 2 (Listing Program Simulasi)

```
function [T Y]=RK4(f,a,h,za)
t=a(1):h:a(2);
N=length(t);
d=length(za);
x=zeros(N,d);
x(1,:)=za;
for i=1:N-1
    k1=h*f(t(i),x(i,:));
    k2=h*f(t(i)+h/2,x(i,:)+k1/2);
    k3=h*f(t(i)+h/2,x(i,:)+k2/2);
    k4=h*f(t(i+1),x(i,:)+k3);
    x(i+1,:)=x(i,:)+(k1+2*k2+2*k3+k4)/6;
end;
T=t';
Y=x;

function y=f1(t,x)
r1=40;
r2=2;
k=100;
a=0.05;
b=0.02;
alpha=0.003;
y(1)=r1*x(1)*(1-(x(1)+x(2))/k)-b*x(1)*x(2)/(x(1)+x(2)+a);
y(2)=r2*x(2)*(1-(x(1)+x(2))/k)+b*x(1)*x(2)/(x(1)+x(2)+a)-alpha*x(2);

clear all;
clc;
[T,Y]=RK4(@f1,[0 100],0.01,[90 10]);
figure(1);
plot(T,Y,'linewidth',3);
xlabel('x(t) uninfected tumor cells');
ylabel('y(t) infected tumor cells');
axis([0 100 0 120]);
title('Plot');
grid on;
hold on;
```

```
% Perhitungan nilai x* dan y*
r1=40;r2=2;k=100;a=0.05;b=0.02;alpha=0.003
y(1)=r1*x(1)*(1-(x(1)+x(2))/k)-
b*x(1)*x(2)/(x(1)+x(2)+a);
y(2)=r2*x(2)*(1-
(x(1)+x(2))/k)+b*x(1)*x(2)/(x(1)+x(2)+a)-
alpha*x(2);
plot(x,y,'r*','LineWidth',1.5);% Plot titik E*
plot(90,10,'k*','LineWidth',1.5);% Plot nilai awal
axis ([0 100 0 120]);
```

