LAMPIRAN

Lampiran 1. Perhitungan Titik Kesetimbangan E_4

Berdasarkan titik kesetimbangan $E_4\left(\gamma, \frac{\delta}{\alpha}, \frac{n_0-\eta\gamma}{\alpha}, \beta n_0 + \frac{\eta\delta}{\alpha}, n_0\right)$ didapatkan matriks Jacobi

$$J = \begin{bmatrix} 0 & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\frac{\eta\delta}{\alpha} & 0 & -\delta & 0 & \frac{\delta}{\alpha} \\ 0 & e_{32} & 0 & 0 & 0 \\ e_{41} & 0 & 0 & 0 & 0 \\ -\mu\beta n_0 & -\mu n_0 & \mu\delta & \mu\gamma & -e_{55} \end{bmatrix}.$$

Untuk memperoleh nilai eigen dari matriks di atas, maka $|J - \lambda I| = 0$, sehingga

$$|J - \lambda I| = \begin{bmatrix} -\lambda & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\frac{\eta\delta}{\alpha} & -\lambda & -\delta & 0 & \frac{\delta}{\alpha} \\ 0 & e_{32} & -\lambda & 0 & 0 \\ e_{41} & 0 & 0 & -\lambda & 0 \\ -\mu\beta n_0 & -\mu n_0 & \mu\delta & \mu\gamma & -e_{55}-\lambda \end{bmatrix} \begin{matrix} b_1 - \frac{\gamma}{\lambda} b_4 \\ - \\ - \\ - \\ b_5 + \frac{\mu\gamma}{\lambda} b_4 \end{matrix}$$

$$= (-\lambda) \begin{bmatrix} -\lambda - \frac{e_{41}\gamma}{\lambda} & \eta\gamma & 0 & \beta\gamma \\ -\frac{\eta\delta}{\alpha} & -\lambda & -\delta & \frac{\delta}{\alpha} \\ 0 & e_{32} & -\lambda & 0 \\ -\mu\beta n_0 + \frac{e_{41}\mu\gamma}{\lambda} & -\mu n_0 & \mu\delta & -e_{55}-\lambda \end{bmatrix} \begin{matrix} - \\ b_2 - \frac{\delta}{\lambda} b_3 \\ - \\ b_4 + \frac{\mu\delta}{\lambda} b_3 \end{matrix}$$

$$= \lambda^2 \begin{bmatrix} -\lambda - \frac{e_{41}\gamma}{\lambda} & \eta\gamma & \beta\gamma \\ -\frac{\eta\delta}{\alpha} & -\lambda - \frac{e_{32}\delta}{\lambda} & \frac{\delta}{\alpha} \\ -\mu\beta n_0 + \frac{e_{41}\mu\gamma}{\lambda} & -\mu n_0 + \frac{e_{32}\mu\delta}{\lambda} & -e_{55}-\lambda \end{bmatrix} = 0,$$

diperoleh

$$|J-\lambda I| = \lambda^2 \Big[\left(-\lambda - \frac{e_{41}\gamma}{\lambda}\right)\left(-\lambda - \frac{e_{32}\delta}{\lambda}\right)(-e_{55}-\lambda) + \eta\gamma\frac{\delta}{\alpha}\left(-\mu\beta n_0 + \frac{e_{41}\mu\gamma}{\lambda}\right) + \beta\gamma\left(-\frac{\eta\delta}{\alpha}\right)\left(-\mu n_0 + \frac{e_{32}\mu\delta}{\lambda}\right) - \left(-\mu\beta n_0 + \frac{e_{41}\mu\gamma}{\lambda}\right)\left(-\lambda - \frac{e_{32}\delta}{\lambda}\right)\beta\gamma - \left(-\mu n_0 + \frac{e_{32}\mu\delta}{\lambda}\right)\frac{\delta}{\alpha}\left(-\lambda - \frac{e_{41}\gamma}{\lambda}\right) - (-e_{55}-\lambda)\left(-\frac{\eta\delta}{\alpha}\right)\eta\gamma\Big] = 0,$$

atau

$$|J-\lambda I| = e_{55}\lambda^4 + \lambda^5 + e_{32}\mu\beta\gamma\delta\lambda^2 + e_{32}\delta\lambda^3 + \frac{e_{41}\mu\gamma\delta}{\alpha}\lambda^2 + e_{41}\gamma\lambda^3 + \frac{\mu\delta n_0}{\alpha}\lambda^3 + e_{32}e_{41}\gamma\delta\lambda + \frac{e_{32}e_{41}\gamma\delta\mu}{\alpha}\lambda + \frac{e_{32}\beta\gamma\eta\delta^2\mu}{\alpha}\lambda + e_{32}\mu\beta^2\gamma\delta n_0\lambda + \mu\beta^2\gamma n_0\lambda^3 + \frac{\delta\gamma\eta^2}{\alpha}\lambda^3 + \frac{e_{55}\delta\gamma\eta^2}{\alpha}\lambda^2 = 0,$$

atau dapat disederhanakan menjadi

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda = 0,$$

dengan

$$a_1 = e_{55},$$

$$a_2 = e_{41}\gamma + e_{32}\delta + \mu\beta^2 n_0\gamma + \frac{\eta^2\delta\gamma}{\alpha} + \frac{\mu n_0\delta}{\alpha},$$

$$a_3 = \frac{e_{41}\gamma\mu\beta\delta}{\alpha} + e_{32}\gamma\mu\beta\delta + \frac{e_{55}\eta^2\delta\gamma}{\alpha},$$

$$a_4 = e_{32}e_{41}\gamma\delta + e_{32}n_0\mu\beta^2\gamma\delta + \frac{e_{41}n_0\gamma\mu\delta}{\alpha} + \frac{e_{32}\delta^2\eta\gamma\mu\beta}{\alpha} - \frac{e_{41}\gamma^2\mu\eta\delta}{\alpha}.$$

Lampiran 2. Perhitungan Titik Kesetimbangan E_7

Matriks Jacobi titik kesetimbangan $E_7(\gamma, f_0, 0, \beta\eta\gamma + \eta f_0, \eta\gamma)$ sebagai berikut

$$J = \begin{bmatrix} 0 & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\eta f_0 & 0 & -\alpha f_0 & 0 & f_0 \\ 0 & 0 & -e_{33} & 0 & 0 \\ e_{41} & 0 & 0 & 0 & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\delta & \mu\gamma & -e_{55} \end{bmatrix}.$$

Untuk memperoleh nilai eigen dari matriks di atas, maka $|J - \lambda I| = 0$, sehingga

$$|J - \lambda I| = \begin{bmatrix} -\lambda & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\eta f_0 & -\lambda & -\alpha f_0 & 0 & f_0 \\ 0 & 0 & -e_{33} - \lambda & 0 & 0 \\ e_{41} & 0 & 0 & -\lambda & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\delta & \mu\gamma & -e_{55} - \lambda \end{bmatrix}$$

$$= (-e_{33} - \lambda) \begin{bmatrix} -\lambda & \eta\gamma & -\gamma & \beta\gamma \\ -\eta f_0 & -\lambda & 0 & f_0 \\ e_{41} & 0 & -\lambda & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\gamma & -e_{55} - \lambda \end{bmatrix} \quad \begin{array}{c} b_1 - \frac{\gamma}{\lambda} b_3 \\ - \\ - \\ b_4 + \frac{\mu\gamma}{\lambda} b_3 \end{array}$$

$$= (\lambda^2 + e_{33}\lambda) \begin{bmatrix} -\lambda - \frac{e_{41}\gamma}{\lambda} & \eta\gamma & \beta\gamma \\ -\eta f_0 & -\lambda & f_0 \\ -\mu\beta\eta\gamma + \frac{e_{41\mu}\gamma}{\lambda} & -\mu\eta\gamma & -e_{55} - \lambda \end{bmatrix} = 0,$$

diperoleh

$$|J - \lambda I| = (\lambda^2 + e_{33}\lambda) \Big[\left(-\lambda - \frac{e_{41}\gamma}{\lambda}\right)(-\lambda)(-e_{55} - \lambda) + \eta\gamma f_0 \left(-\mu\beta\eta\gamma + \frac{e_{41\mu}\gamma}{\lambda}\right) + \beta\gamma(-\eta f_0)(-\mu\eta\gamma) - \left(-\mu\beta\eta\gamma + \frac{e_{41\mu}\gamma}{\lambda}\right)(-\lambda)\beta\gamma - (-\mu\eta\gamma) f_0 \left(-\lambda - \frac{e_{41}\gamma}{\lambda}\right) - (-e_{55} - \lambda)(-\eta f_0)\eta\gamma \Big] = 0,$$

atau

$$|J-\lambda I| = e_{55}\lambda^4 + \lambda^5 + e_{32}\mu\gamma f_0\lambda^2 + e_{41}\gamma\lambda^3 + \mu\beta^2\eta\gamma^2\lambda^2 + \mu\eta\gamma f_0\lambda^3 + e_{55}\eta^2 f_0\gamma\lambda^2 + \eta^2 f_0\gamma\lambda^3 + e_{33}e_{55}\lambda^3 + \frac{e_{32}\beta\gamma\eta\delta^2\mu}{\alpha}\lambda + e_{55}\lambda^4 + e_{33}e_{41}\mu\gamma f_0\lambda + e_{33}e_{41}\gamma\lambda^2 + e_{33}\mu\beta^2\eta\gamma^2\lambda^2 + e_{33}\mu\eta\gamma f_0\lambda^2 + e_{33}e_{55}\eta^2 f_0\gamma\lambda + e_{33}\eta^2 f_0\gamma\lambda^2 = 0,$$

atau dapat disederhanakan menjadi

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda = 0,$$

dengan

$$a_1 = e_{55+}e_{33},$$

$$a_2 = e_{33}e_{55} + e_{41}\gamma + \mu\eta\gamma f_0 + \gamma f_0 + \mu\beta^2\eta\gamma^2,$$

$$a_3 = e_{33}\mu\eta\gamma f_0 + e_{33}\eta^2\gamma f_0 + e_{55}\eta^2\gamma f_0 + e_{33}e_{41}\gamma + e_{41}\mu\gamma f_0 + e_{33}\mu\beta^2\eta\gamma^2,$$

$$a_4 = e_{33}e_{55}\eta^2\gamma f_0 + e_{33}e_{41}\mu\gamma f_0.$$

Lampiran 3. Perhitungan Titik Kesetimbangan E_8

Matriks Jacobi titik kesetimbangan $E_8\left(\gamma, \frac{p_0}{\eta} - \beta\gamma, 0, p_0, \eta\gamma\right)$ sebagai berikut

$$J = \begin{bmatrix} 0 & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\eta e_{25} & 0 & -\alpha e_{25} & 0 & e_{25} \\ 0 & 0 & -e_{33} & 0 & 0 \\ p_0 & 0 & 0 & 0 & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\delta & \mu\gamma & -\dfrac{\mu p_0}{\eta} \end{bmatrix}.$$

Untuk memperoleh nilai eigen dari matriks di atas, maka $|J - \lambda I| = 0$, sehingga

$$|J - \lambda I| = \begin{bmatrix} -\lambda & \eta\gamma & 0 & -\gamma & \beta\gamma \\ -\eta e_{25} & -\lambda & -\alpha e_{25} & 0 & e_{25} \\ 0 & 0 & -e_{33} - \lambda & 0 & 0 \\ p_0 & 0 & 0 & -\lambda & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\delta & \mu\gamma & -\dfrac{\mu p_0}{\eta} - \lambda \end{bmatrix}$$

$$(-e_{33} - \lambda)\begin{bmatrix} -\lambda & \eta\gamma & -\gamma & \beta\gamma \\ -\eta e_{25} & -\lambda & 0 & e_{25} \\ p_0 & 0 & -\lambda & 0 \\ -\mu\beta\eta\gamma & -\mu\eta\gamma & \mu\gamma & -\dfrac{\mu p_0}{\eta} - \lambda \end{bmatrix} \begin{matrix} b_1 - \dfrac{\gamma}{\lambda} b_3 \\ - \\ - \\ b_4 + \dfrac{\mu\gamma}{\lambda} b_3 \end{matrix}$$

$$(\lambda^2 + e_{33}\lambda)\begin{bmatrix} -\lambda - \dfrac{p_0\gamma}{\lambda} & \eta\gamma & \beta\gamma \\ -\eta e_{25} & -\lambda & e_{25} \\ -\mu\beta\eta\gamma + \dfrac{p_0\mu\gamma}{\lambda} & -\mu\eta\gamma & -\dfrac{\mu p_0}{\eta} - \lambda \end{bmatrix} = 0,$$

sehingga diperoleh determinan matriks sebagai berikut

$$|J - \lambda I| = (\lambda^2 + e_{33}\lambda)\left[\left(-\lambda - \frac{p_0\gamma}{\lambda}\right)(-\lambda)\left(-\frac{\mu p_0}{\eta} - \lambda\right) + \right.$$
$$\eta\gamma e_{25}\left(-\mu\beta\eta\gamma + \frac{p_0\mu\gamma}{\lambda}\right) + \beta\gamma(-\eta e_{25})(-\mu\eta\gamma) -$$

$$\left(-\mu\beta\eta\gamma + \frac{p_0\mu\gamma}{\lambda}\right)(-\lambda)\beta\gamma - (-\mu\eta\gamma)e_{25}\left(-\lambda - \frac{p_0\gamma}{\lambda}\right) - \left(-\frac{\mu p_0}{\eta} - \lambda\right)(-\eta e_{25})\eta\gamma\Big] = 0,$$

atau

$$|J - \lambda I| = \frac{\mu p_0}{\eta}\lambda^4 + \lambda^5 + e_{25}\mu\gamma\lambda^2 + p_0\gamma\lambda^3 + \mu\beta^2\eta\gamma^2\lambda^3 + e_{25}\eta\mu\gamma\lambda^3 + e_{25}\gamma\eta\mu p_0\lambda^2 + e_{25}\eta^2\lambda^3 + \frac{e_{33}\mu p_0}{\eta}\lambda^3 + e_{33}\lambda^4 + e_{33}e_{25}\gamma\mu p_0\lambda + e_{33}\gamma p_0\lambda^2 + e_{33}\eta\mu\gamma^2\beta^2\lambda^2 + e_{33}e_{25}\eta\mu\gamma\lambda^2 + e_{25}e_{33}\gamma\eta\mu p_0\lambda + e_{25}e_{33}\eta^2\gamma\lambda^2 = 0,$$

atau dapat disederhanakan menjadi

$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda = 0,$

dengan

$a_1 = e_{33} + \frac{\mu p_0}{\eta},$
$a_2 = e_{25}\eta\mu\gamma + p_0\gamma + \mu\eta\beta^2\gamma^2 + e_{25}\eta^2\gamma + \frac{e_{33}\mu p_0}{\eta},$
$a_3 = e_{33}\gamma p_0 + e_{33}e_{25}\eta\mu\gamma + e_{25}\gamma\eta\mu p_0 + e_{25}\mu\gamma p_0 + e_{33}\eta\mu\gamma^2\beta^2 + e_{25}e_{33}\eta^2\gamma,$
$a_4 = e_{33}e_{25}\gamma\mu p_0 + e_{25}e_{33}\gamma\eta\mu p_0.$